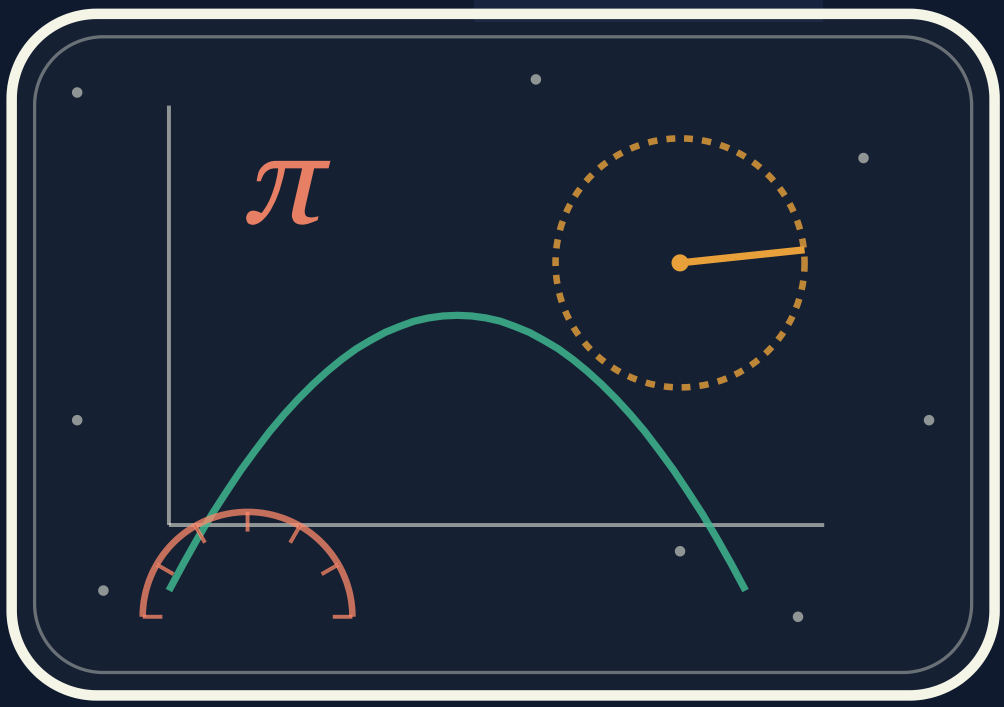
AQA GCSE MATHEMATICS (8300)

Maths Revision Guide

Number, Algebra, Ratio, Geometry, Probability and Statistics

Seventeen topics on two pages each: learn it, then use it. Every worked answer marked exactly as AQA marks it, M1 for method and A1 for accuracy, plus a practice paper to finish.



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Start here

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Two pages a topic

Learn it: sixty seconds, the facts with a circle to colour, key words, and a Spot it question.

Use it: a worked example marked as AQA marks it (M1 method, A1 accuracy, B1 independent), how it could be asked, a memory box, and a quick-fire quiz.

Four steps, every time

Predict how much you'll remember. **Recall** with the page covered. **Check** in another colour. **Repair** the weakest part, then return in two days.

Where it comes from

Facts follow AQA's specification (8300, v1.1). Exam advice comes from AQA's papers, mark schemes and examiners' reports (June 2022, June 2023). No AQA question or mark scheme is reproduced: every worked example, Spot it and the practice paper were written for this guide.

What HT means

All three papers can test any part of the spec; this guide is built for Higher. A red HT marks a fact the spec lists as Higher only. An untagged fact is common to both tiers.

A five-week plan

Weeks 1-2: Number and Algebra (1-7). **Week 3:** Ratio and angle work (8-11). **Week 4:** the rest of Geometry, Probability, Statistics (12-17). **Week 5:** formulae, calculator skills, examiners' notes, then the practice paper. Return to each topic two days later and a week later; the tracker is on page 49.

First edition. No serving maths teacher has read this guide through yet. If something should change, tell us at info@empowereducators.co.uk.

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Know the exam

All three papers on one page

Every paper	
Content	Any part of the spec can be tested on any paper: this is not a 'Paper 1' guide
Exam	Written, 1 h 30 min, 80 marks each
Weighting	33⅓% of the GCSE each, 240 marks in total
Tiers	Foundation (1 to 5) or Higher (4 to 9); this guide is Higher
Calculator	Paper 1: non-calculator. Papers 2, 3: calculator

Bring a pencil, ruler, compasses and protractor to every paper, plus a scientific calculator for Papers 2 and 3. Black ink, pencil for diagrams. Show your working, and keep the Formulae Sheet close by.

What a paper looks like

Each paper mixes short, single-mark questions with longer, multi-step problems, demand rising through the paper. The first few questions are usually the most accessible.

AO1 (recall) is about 40% of the marks at Higher tier; AO2 (reasoning) and AO3 (problem-solving) share the other 60%. More than half the marks need you to **use** maths, not just state it.

Every topic can appear on every paper: a circle theorem question is just as likely on a calculator paper as the non-calculator one.

What the question word asks for

Word	AQA's meaning, shortened	What it looks like
Work out · Calculate · Find	Do a calculation and give the answer.	Show every step; the answer line alone may not score full marks.
Solve	Find the value(s) that make an equation or inequality true.	x = … , or the solution set for an inequality.
Write down	A short answer, no working expected.	"7." or "x > 4."
Show that	Prove a given result, showing every step.	Work down the page in order; the method earns the marks.
Simplify · Factorise · Expand	Carry out the process; give the simplest form.	Simplifying is always expected, with or without being told.
Give a reason · Explain	Justify a statement mathematically, not just in words.	Name a value, a rule or a theorem, not 'because it looks like it'.
Describe fully	Name a single transformation, with everything it needs.	"A reflection in the line x = 2," not just "a reflection."
Prove	Show a general statement is always true, algebraically.	Use a letter such as n; one example is never a proof.
Hence, or otherwise,	Use a previous answer (hence), or any valid method (otherwise).	The hence route is usually faster; either can score full marks.
Circle · Tick	Select the correct option given.	No working needed, but a second, wrong choice can cost the mark.
Estimate	Round first, then calculate, for an approximate answer.	Round to 1 significant figure unless told otherwise.
Plot · Draw · Complete	Mark data on a graph, or finish a diagram or table.	Pencil and ruler; one smooth curve, not short joined lines.

Working, and 'show that' questions

Method (M) marks are for a correct method, even with a later slip. Accuracy (A) marks need a correct method *and* a correct value. Independent (B) marks don't depend on a particular method.

Practise this: two worked, multi-step problems marked exactly this way are on pages 42 and 43.

Sources: AQA 8300 specification (version 1.1); AQA GCSE Mathematics papers, mark schemes and Reports on the Examination (June 2022, June 2023).

Part one · Number

1 Number: structure and calculation

Learn it

In sixty seconds

Order and calculate with integers, decimals and fractions using the four operations and the correct order of operations. Write any number as a product of its prime factors to find the HCF and LCM. Use the laws of indices, and at Higher tier, fractional indices and surds. Standard form writes very large or very small numbers as $A \times 10^n$, where $1 \leq A < 10$.

Facts to know

Try it, then colour the circle: ● knew it ◐ shaky ○ missed

- ○ Order positive and negative integers, decimals and fractions using <, >, ≤, ≥ and a number line.
- ○ Add, subtract, multiply and divide positive and negative integers, decimals and fractions (proper, improper and mixed), using formal written methods.
- ○ Follow the order of operations: **brackets**, then **powers and roots**, then **multiplication and division**, then **addition and subtraction**. Use inverse operations (the opposite operation) to check an answer.
- ○ Write a number as a **product of its prime factors** in index form ($54 = 2 \times 3^3$). Use the prime factors of two numbers to find their **highest common factor** (HCF) and **lowest common multiple** (LCM).
- ○ Use systematic listing (tables, lists, diagrams) to count outcomes without missing any or repeating any. HT The **product rule for counting**: multiply the number of choices at each independent stage, rather than add them.
- ○ Know the square numbers to 15×15 and the cube numbers to 5^3. Use positive integer powers and their roots (square, cube and higher). HT Estimate the power or root of any positive number by comparing it with nearby powers you do know.
- ○ Use the **laws of indices** for integer powers: $a^m \times a^n = a^{m+n}$; $a^m \div a^n = a^{m-n}$; $(a^m)^n = a^{mn}$; $a^0 = 1$; $a^{-n} = 1 \div a^n$.
- ○ HT Calculate with **fractional indices**: $a^{1/n}$ is the n^{th} root of a, and $a^{m/n}$ is the n^{th} root of a, raised to the power m.
- ○ HT Simplify **surd** expressions, such as $\sqrt{12} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$, and **rationalise a denominator** such as $1/\sqrt{2} = \sqrt{2}/2$, by multiplying top and bottom by the same surd.
- ○ Calculate exactly with fractions and with multiples of π, without rounding at any stage. Write and calculate with **standard form**, $A \times 10^n$ where $1 \leq A < 10$, with and without a calculator, and read it correctly off a calculator display.

Key words

- **Prime number** a whole number greater than 1 with exactly two factors: 1 and itself. ☐
- **Factor** a whole number that divides exactly into another. ☐
- **Multiple** a number in another number's times table. ☐
- **HCF** highest common factor: the biggest number that divides exactly into two or more numbers. ☐
- **LCM** lowest common multiple: the smallest number that two or more numbers divide into exactly. ☐
- **Surd** a root, such as $\sqrt{3}$, that cannot be simplified to a whole number or exact fraction. ☐
- **Standard form** a number written as $A \times 10^n$, where $1 \leq A < 10$. ☐
- **Reciprocal** 1 divided by a number; the reciprocal of a/b is b/a. ☐

The product rule for counting. A café has 4 starters and 3 mains. The number of different starter-and-main combinations is $4 \times 3 = 12$, not $4 + 3$: for each of the 4 starters there are 3 choices of main. Multiply the number of options at each independent stage, however many stages there are.

When dividing two numbers in standard form, don't cancel digits across the two parts: divide the front numbers and subtract the powers of 10 separately. Converting both numbers out of standard form first rarely finishes in time. When multiplying powers of the same base, only the indices add: a common slip is to multiply the bases as well, so $3^5 \times 3^2$ becomes 9^7 instead of 3^7.

Spot it Which law is right?

Which of these is correctly simplified?

(A) $3^5 \times 3^2 = 9^7$ (B) $3^5 \times 3^2 = 3^7$

(C) $3^5 \times 3^2 = 3^{10}$ (D) $3^5 \times 3^2 = 6^7$

Cover this, choose, then read

B. Keep the base as 3 and add the indices: 5 + 2 = 7. The other options multiply the bases, multiply the indices, or add the bases: none of those is the rule.

Part one · Number

1 Number: structure and calculation

Use it

54 and 90, broken into prime factors

54	90
54 = 2 × 27 = 2 × 3 × 9 = 2 × 3 × 3 × 3 **$54 = 2 \times 3^3$**	90 = 2 × 45 = 2 × 9 × 5 = 2 × 3 × 3 × 5 **$90 = 2 \times 3^2 \times 5$**

HCF: take the **lowest** power of each factor common to both: $2^1 \times 3^2 = 18$. LCM: take the **highest** power of every factor that appears in either number: $2^1 \times 3^3 \times 5^1 = 270$.

HT Find the HCF and the LCM of 54 and 90, by first writing each as a product of prime factors. [4 marks]

$54 = 2 \times 3^3$ ✓. $90 = 2 \times 3^2 \times 5$ ✓. HCF $= 2 \times 3^2 = 18$ ✓. LCM $= 2 \times 3^3 \times 5 = 270$ ✓.

4/4

M1 for a correct prime factorisation of 54. M1 for a correct prime factorisation of 90. A1 for the HCF, using the lowest power of each shared factor. A1 for the LCM, using the highest power of every factor present. A factor tree or repeated division both earn the M1: the mark scheme doesn't care which method reaches the factorisation.

It could be asked as

- Write a number as a product of its prime factors, and use it to find an HCF or LCM.
- Simplify or evaluate an expression using the laws of indices, including negative or fractional indices.
- HT Simplify a surd, or rationalise a denominator that contains a surd.
- Convert between standard form and ordinary numbers, and calculate with standard form on a non-calculator paper.

Cover both pages and write your sixty seconds from memory

predict ___ / 10 got ___ / 10

Quick-fire. Answers are on page 48

1.1 Write 60 as a product of its prime factors.

1.2 Work out 4^{-2}.

1.3 HT Work out $8^{2/3}$.

1.4 Write 0.00047 in standard form.

Part one · Number

2 Fractions, decimals, percentages and accuracy

Learn it

In sixty seconds

Fractions, decimals and percentages are different ways of writing the same value, and you should move between them confidently. Round sensibly, and don't round until the final step of a calculation. A rounded or measured value has an error interval: the true value could be anywhere between a lower and an upper bound.

Facts to know

Try it, then colour the circle: ● knew it ◐ shaky ○ missed

- ○ Convert between terminating decimals and their matching fraction (3.5 = 7/2, 0.375 = 3/8) and order them alongside percentages and fractions on the same scale.
- ○ HT Convert a **recurring decimal** to its exact fraction and back (0.277... = 5/18) using an algebraic method: let x equal the decimal, multiply to shift the repeating block, then subtract.
- ○ Interpret a fraction or a percentage as an **operator**: finding 3/8 of an amount or 15% of an amount both mean multiply by that operator (0.375, or 0.15).
- ○ Estimate an answer, and check a calculation using approximation (round each number to 1 significant figure first, then calculate).
- ○ **Round** to a given number of decimal places or significant figures. Never round values during the middle steps of a calculation: round only the final answer, to the accuracy asked for.
- ○ Use inequality notation to write an **error interval** caused by rounding or truncation, such as $7.35 \leq x < 7.45$ for a value given as 7.4 to 1 decimal place.
- ○ HT Apply and interpret **upper and lower bounds**: for a measurement rounded to the nearest unit, the bound is half that unit either side of the rounded value.
- ○ Use standard units of mass, length, time, money and other measures, including compound measures, with decimal quantities, and know the metric conversion factors for length, area, volume and capacity. Imperial to metric conversions are always given in the question.
- ○ Know the vocabulary of household finance: profit, loss, cost price, selling price, debit, credit, balance, income tax, VAT and interest rate.
- ○ Calculator On Papers 2 and 3, use a calculator efficiently: the memory function, brackets, and the fraction and power keys, so nothing is rounded before the final answer.

Key words

- **Terminating decimal** a decimal that ends, rather than repeating forever. ☐
- **Recurring decimal** a decimal with a digit or block of digits that repeats forever. ☐
- **Significant figure** a digit that carries meaning about the size of a number, counted from the first non-zero digit. ☐
- **Truncation** cutting a number short without rounding it. ☐
- **Error interval** the range of values a rounded or truncated number could actually be. ☐
- **Lower bound** the smallest value a rounded measurement could have been before rounding. ☐
- **Upper bound** the smallest value that would have rounded up to the next figure. ☐
- **Operator** a fraction or percentage used as a multiplier to act on a quantity. ☐

Why bounds use half the rounding unit. A length given as 7.4 cm to 1 decimal place could be any true value that rounds to 7.4: that's everything from 7.35 cm up to (but not including) 7.45 cm, since 7.45 would round up to 7.5. The upper bound is an open inequality (<) for exactly this reason.

A 2023 Paper 2 question asked students to use bounds in a calculation: a significant number did not consider bounds at all, and a common error was to pair the wrong bounds together, such as using the upper bound of one measurement with the lower bound of the other when the calculation needed both upper, or both lower. Decide whether you need the largest possible answer or the smallest possible answer, then choose both bounds to match.

Spot it Which bound?

A mass is given as 350 g, to the nearest 10 g. What is the lower bound?

(A) 350 g (B) 340 g
(C) 345 g (D) 355 g

Cover this, choose, then read

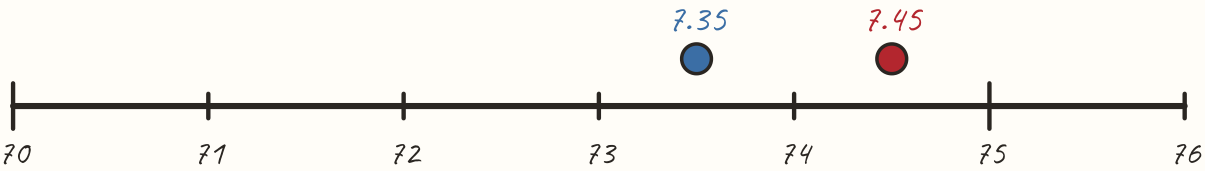
C. The rounding unit is 10 g, so the bound is half of that (5 g) either side of 350 g: 345 g to 355 g. 340 g uses a whole 10 g instead of half; 355 g is the upper bound, not the lower.

Part one · Number

2 Fractions, decimals, percentages and accuracy

Use it

Error interval for a length given as 7.4 cm (1 d.p.)



Every value from 7.35 cm up to, but not including, 7.45 cm would round to 7.4 cm. Write this as $7.35 \leq x < 7.45$.

HT A rectangular tile has a length of 7.4 cm and a width of 3.6 cm, both given to 1 decimal place. Work out the upper bound for the area of the tile. [3 marks]

Upper bound of length = 7.45 cm ✓. Upper bound of width = 3.65 cm ✓. Area = 7.45 × 3.65 = 27.1925 ✓, so the upper bound is 27.1925 cm² (or 27.19 cm² to 2 d.p.).

3/3

M1 for the upper bound of the length, 7.45. M1 for the upper bound of the width, 3.65. A1 for multiplying the two upper bounds together: the biggest area needs the biggest length and the biggest width, not a mix of bounds.

It could be asked as

- Convert between a fraction, a decimal and a percentage, including a recurring decimal.
- Round a number, or use approximation to estimate the answer to a calculation.
- HT Write an error interval, or find an upper or lower bound and use it in a calculation.
- Use a calculator to carry out a multi-step calculation without rounding early.

Cover both pages and write your sixty seconds from memory

predict ___ / 10 got ___ / 10

Quick-fire. Answers are on page 48

2.1 Write 0.6 as a fraction in its simplest form.

2.2 Round 4.8471 to 2 significant figures.

2.3 A length is given as 12 cm to the nearest cm. Write its error interval.

2.4 Estimate the value of 61 × 0.21 by rounding each number to 1 significant figure first.

Part two · Algebra

3 Expressions, formulae and identities

Learn it

In sixty seconds

Algebraic notation follows fixed conventions: ab means a × b, and a^2 means a × a. Expand, simplify and factorise expressions by collecting like terms, multiplying out brackets and taking out common factors. A formula connects variables and can be rearranged to change its subject. An identity is true for every value of the letters, not just some.

Facts to know

Try it, then colour the circle: ● knew it ◐ shaky ○ missed

- ○ Algebraic notation: *ab* means a × b; *3y* means y + y + y; a^2 means a × a; *a/b* means a ÷ b. Coefficients are written as fractions, not decimals, and answers are given in their simplest form.
- ○ Substitute numerical values into formulae and expressions, including scientific or financial formulae from other subjects.
- ○ Know the vocabulary of expressions, equations, formulae, inequalities, terms and factors. An **equation** has an equals sign and can be solved; an **identity** (≡) is true for every value of the letters.
- ○ **Simplify** by collecting like terms, multiplying a single term over a bracket, and taking out common factors. **Expand** two binomials, such as (x + 3)(x + 5). **Factorise** a quadratic of the form $x^2 + bx + c$, including the difference of two squares ($x^2 - 9 = (x + 3)(x - 3)$).
- ○ HT Expand three or more binomials, and factorise a quadratic of the general form $ax^2 + bx + c$ where $a \neq 1$. Simplify and manipulate **algebraic fractions**, including adding, subtracting, multiplying and dividing them.
- ○ **Rearrange a formula** to change its subject, such as making x the subject of y = (3x − 2) / 4.
- ○ Interpret a simple expression as a **function**: an input-output machine, such as f(x) = 2x + 1.
- ○ HT Use **inverse function** notation $f^{-1}(x)$ for the reverse process, and **composite function** notation fg(x) for doing g first, then f, to the result.
- ○ HT Use algebra to **construct a mathematical argument or proof** that two expressions are always equal, or that a statement is always true, for example that the sum of three consecutive integers is always a multiple of 3.
- ○ Know and use standard formulae from the Appendix (such as the quadratic formula and the area of a trapezium), and formulae from other subjects, given in words or using symbols.

Key words

- ☐ **Expression** a collection of terms with no equals sign, such as 3x + 2.
- ☐ **Equation** a statement with an equals sign that is true for only some, or no, values of the letter.
- ☐ **Identity** a statement that is true for every value of the letters, shown with the symbol ≡.
- ☐ **Term** a single number, letter, or the product of numbers and letters, in an expression.
- ☐ **Factorise** write an expression as a product of its factors, using brackets.
- ☐ **Subject** the letter on its own on one side of a formula, such as y in y = mx + c.
- ☐ **Function** a rule that takes an input and gives exactly one output.
- ☐ **Composite function** the result of applying one function, then another, to the same input.

An equation versus an identity. 2x + 3 = 7 is an equation: it is only true when x = 2. $(x + 1)^2 \equiv x^2 + 2x + 1$ is an identity: it is true for every value of x, which is why it uses ≡ rather than =. Proving an identity usually means expanding one side until it matches the other, term by term.

A 2023 paper asked students to expand and simplify 5(3x + 4) - 2(x - 1): a common error was to expand the second bracket wrongly, turning -2(x - 1) into -2x - 2 instead of the correct -2x + 2, so check the sign is distributed to every term inside the bracket, not just the first one. Composite functions are also a known weak spot: a frequent mistake is to multiply the two functions together instead of substituting one function into the other.

Spot it Which expansion is right?

Which of these shows (x + 4)(x − 3) expanded and simplified correctly?

(A) $x^2 + x - 12$ (B) $x^2 - x - 12$

(C) $x^2 + 7x - 12$ (D) $x^2 + x + 12$

Cover this, choose, then read

A. x × x = x^2, x × -3 = -3x, 4 × x = 4x, 4 × -3 = -12. Combine -3x + 4x to get +x, giving $x^2 + x - 12$. The other options have slipped a sign or added the constants instead of multiplying them.

Part two · Algebra

3 Expressions, formulae and identities

Use it

Function machines: f(x), the inverse, and the composite

f(x) = 2x + 1	HT f⁻¹(x), the inverse
Input 5 → ×2 → +1 → Output 11. **f(5) = 11**	Undo in reverse order: -1 then ÷2. **f⁻¹(x) = (x − 1) ÷ 2**

HT

Composite fg(x) means do g first, then put the result into f. If g(x) = x + 3, then fg(x) = f(x + 3) = 2(x + 3) + 1 = 2x + 7.

Make x the subject of the formula y = (3x − 2) / 4. [3 marks]

4y = 3x − 2 ✓ (multiply both sides by 4). 4y + 2 = 3x ✓ (add 2 to both sides). x = (4y + 2) / 3 ✓ (divide both sides by 3).

3/3

M1 for multiplying both sides by 4 to clear the fraction. M1 for adding 2 to both sides to isolate the 3x term. A1 for dividing by 3 to reach x on its own: the whole right-hand side must be divided by 3, not just part of it.

It could be asked as

- Expand and simplify brackets, or factorise a quadratic expression.
- Rearrange a formula to make a given letter the subject.
- HT Work with function notation, including an inverse or a composite function.
- HT Use algebra to prove that a statement is always true.

Cover both pages and write your sixty seconds from memory

predict ___ / 10 got ___ / 10

Quick-fire. Answers are on page 48

3.1 Expand and simplify 3(2x + 1) − 2(x − 4).

3.2 Factorise $x^2 - 16$.

3.3 Make r the subject of $A = \pi r^2$.

3.4 HT If $f(x) = x^2$ and g(x) = x + 1, work out fg(2).

Part two · Algebra

4 Straight-line graphs

Learn it

In sixty seconds

Any straight line can be written as y = mx + c, where m is the gradient (how steep the line is) and c is the y-intercept (where it crosses the y-axis). Parallel lines have the same gradient. Perpendicular lines meet at a right angle, and their gradients multiply to -1.

Facts to know

Try it, then colour the circle: ● *knew it* ◐ *shaky* ○ *missed*

- ○ Work with coordinates in all four quadrants, including negative x and y values.
- ○ Plot a straight-line graph from an equation such as y = mx + c, or from a table of values, using at least three points as a check.
- ○ Identify and interpret the **gradient** (steepness and direction) and the **y-intercept** of a linear function, both from a graph and from its equation y = mx + c.
- ○ Use the form y = mx + c to identify **parallel** lines: two lines are parallel if and only if they have the same gradient, m.
- ○ HT Use the form y = mx + c to identify **perpendicular** lines: two lines are perpendicular if the product of their gradients is -1, so a line with gradient m is perpendicular to one with gradient -1/m.
- ○ Find the **gradient** between two points (x_1, y_1) and (x_2, y_2) using $(y_2 - y_1) \div (x_2 - x_1)$.
- ○ Find the equation of the line through two given points, or through one point with a given gradient, by substituting into y = mx + c and solving for c.
- ○ Interpret the gradient of a straight-line graph as a **rate of change**, such as speed on a distance-time graph, in a real context.
- ○ Plot and interpret real-life graphs of non-standard functions, such as simple kinematic (distance, speed, time) problems, to find approximate solutions.
- ○ HT Recognise and use the equation of a circle with centre the origin, $x^2 + y^2 = r^2$, and find the equation of a tangent to a circle at a given point.

Key words

Gradient a measure of how steep a line is: the change in y divided by the change in x.	☐
y-intercept the point where a line or curve crosses the y-axis, where x = 0.	☐
Parallel lines that never meet and have the same gradient.	☐
Perpendicular lines that cross at a right angle (90°).	☐
Linear a graph or equation that produces a straight line.	☐
Quadrant one of the four regions of a coordinate grid, divided by the x- and y-axes.	☐
Rate of change how one quantity changes as another changes, shown by the gradient.	☐
Tangent a straight line that touches a curve at exactly one point.	☐

Reading the gradient and intercept straight off the equation. In y = 3x + 2, the gradient is 3 (for every 1 across, the line goes 3 up) and the y-intercept is 2 (where x = 0). If the equation isn't in that form, such as 2y = 6x − 4, divide every term by 2 first to get y = 3x − 2, and only then read off m and c.

A 2023 Paper 1 question gave one point on a line and a gradient, and asked for a related point using a right angle: about half of students answered fully correctly, but a number got stuck after finding just one coordinate. Flip the fraction and change the sign: a gradient of 2/3 becomes -3/2 for the perpendicular.

Spot it Which line is parallel?

Which of these lines is parallel to y = 4x − 1?

(A) y = x − 1
(B) y = -4x − 1
(C) y = 7x + 4
(D) y = 4x + 7

Cover this, choose, then read

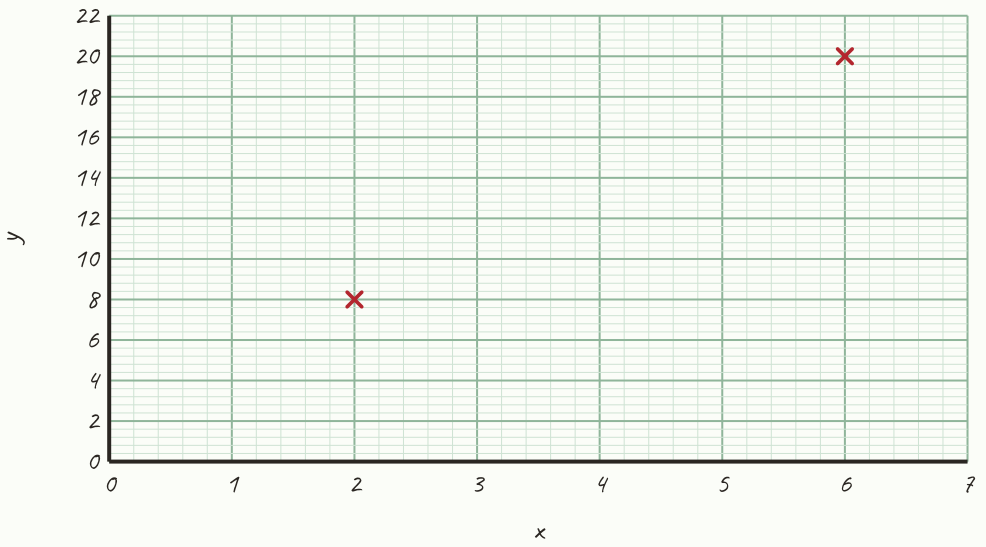
D. Parallel lines share the same gradient. y = 4x − 1 has gradient 4, and only y = 4x + 7 also has gradient 4; a different y-intercept doesn't matter. The others have gradient 1, -4 and 7.

Part two · Algebra

4 Straight-line graphs

Use it

The line through (2, 8) and (6, 20)



Gradient = (20 − 8) ÷ (6 − 2) = 3. Substituting (2, 8): 8 = 3 × 2 + c, so c = 2. The line is y = 3x + 2.

A line passes through (2, 8) and (6, 20). Find the equation of the line, and state the gradient of a line perpendicular to it. [4 marks]

Gradient = (20 − 8) ÷ (6 − 2) = 3 ✓. Using y = mx + c and (2, 8): 8 = 3(2) + c, so c = 2 ✓. Equation: y = 3x + 2 ✓.

HT Perpendicular gradient = -1 ÷ 3 = -1/3 ✓.

4/4

M1 for a correct method for the gradient. A1 for the correct gradient, 3. A1 for the full equation, y = 3x + 2, found by substituting a point to get c. HT *B1 (Higher) for the perpendicular gradient, found by flipping the fraction and changing the sign.*

It could be asked as

- Find the gradient and the equation of a line through two given points.
- HT Decide whether two lines are parallel or perpendicular, using their gradients.
- Interpret a real-life graph, such as a distance-time graph, including its gradient.
- HT Find the equation of a tangent to a circle at a given point.

Cover both pages and write your sixty seconds from memory

predict ___ / 10 got ___ / 10

Quick-fire. Answers are on page 48

4.1 State the gradient and y-intercept of y = -2x + 5.

4.2 Are y = 5x − 3 and y = 5x + 1 parallel?

4.3 HT State the gradient of a line perpendicular to y = 2x + 1.

4.4 Find the equation of the line through (0, 4) with gradient 3.

Part two · Algebra

5 Quadratic and other graphs

Learn it

In sixty seconds

A quadratic graph is a symmetrical curve called a parabola. Its roots are where it crosses the x-axis, and its turning point can be found by completing the square. Recognise the shapes of cubic, reciprocal, exponential and trigonometric graphs too, and how transformations move or reflect them.

Facts to know

Try it, then colour the circle: ● knew it ◐ shaky ○ missed

- ○ Recognise, sketch and interpret the graph of a **quadratic function**: a symmetrical curve (a parabola), with a minimum point if positive x^2 or a maximum point if negative x^2.
- ○ Identify and interpret the **roots** (where the curve crosses the x-axis, y = 0), the y-intercept, and the **turning point** of a quadratic graph.
- ○ Deduce the roots of a quadratic **algebraically**, by factorising.
  HT Deduce the turning point by **completing the square**: $x^2 + bx + c = (x + b/2)^2 + (c - b^2/4)$.
- ○ Recognise and sketch the graph of a simple **cubic function** and the **reciprocal function** $y = 1/x$ ($x \neq 0$), including its two separate branches and asymptotes.
- ○ HT Recognise and sketch **exponential** graphs $y = k^x$ for positive k, and the **trigonometric** graphs y = sin x, y = cos x and y = tan x, for any angle.
- ○ HT Sketch a **translation** or **reflection** of a given function, such as y = f(x) + a (vertical shift) or y = f(-x) (reflection in the y-axis).
- ○ Plot and interpret graphs of real-life, non-standard functions to find approximate solutions, such as a quantity that grows or decays over time.
- ○ HT Calculate or estimate the **gradient of a curve** at a point, by drawing a tangent, and the **area under a curve**, by splitting it into trapezia and triangles.
- ○ Find approximate solutions to an equation using a graph: where a line or curve crosses another, or crosses the x-axis.
- ○ HT Find an approximate solution to an equation such as $x^3 - x - 3 = 0$ numerically, using **iteration** with a given recursive formula and suffix notation ($x_{n+1} = ...$).

Key words

- **Parabola** the symmetrical curve shape of a quadratic graph. ☐
- **Root** a value of x where a graph crosses the x-axis (y = 0). ☐
- **Turning point** the minimum or maximum point of a curve, where its gradient is momentarily zero. ☐
- **Completing the square** rewriting $x^2 + bx + c$ as $(x + b/2)^2$ plus a constant. ☐
- **Asymptote** a line a curve gets closer and closer to, but never touches. ☐
- **Iteration** repeating a calculation, feeding each answer back in as the next input. ☐
- **Tangent to a curve** a straight line that touches the curve at one point, used to estimate a gradient. ☐
- **Exponential** a graph of the form $y = k^x$, which grows (or decays) by the same factor every step. ☐

Why completing the square finds the turning point. Writing $x^2 - 6x + 5$ as $(x - 3)^2 - 4$ shows the minimum value of $(x - 3)^2$ is 0, when x = 3, so the smallest the whole expression can be is -4. The turning point is (3, -4): the number inside the bracket (with its sign flipped) gives the x-coordinate, and the number left outside gives the y-coordinate.

A 2022 question asked students to complete the square: common errors included halving the coefficient of x incorrectly (working out 9^2 as 18, or using $(x + 9)^2$ instead of $(x + 4.5)^2$) and adding the square of the wrong number back on. Halve the coefficient of x first, write that number inside the bracket, then work out what the bracket actually gives you before subtracting it to balance the expression.

Spot it Which sketch is right?

$y = x^2 - 6x + 5$ crosses the x-axis at 1 and 5. Where is its turning point?

(A) (3, 4), a maximum
(B) (-3, -4), a minimum
(C) (3, -4), a minimum
(D) (6, -4), a minimum

Cover this, choose, then read

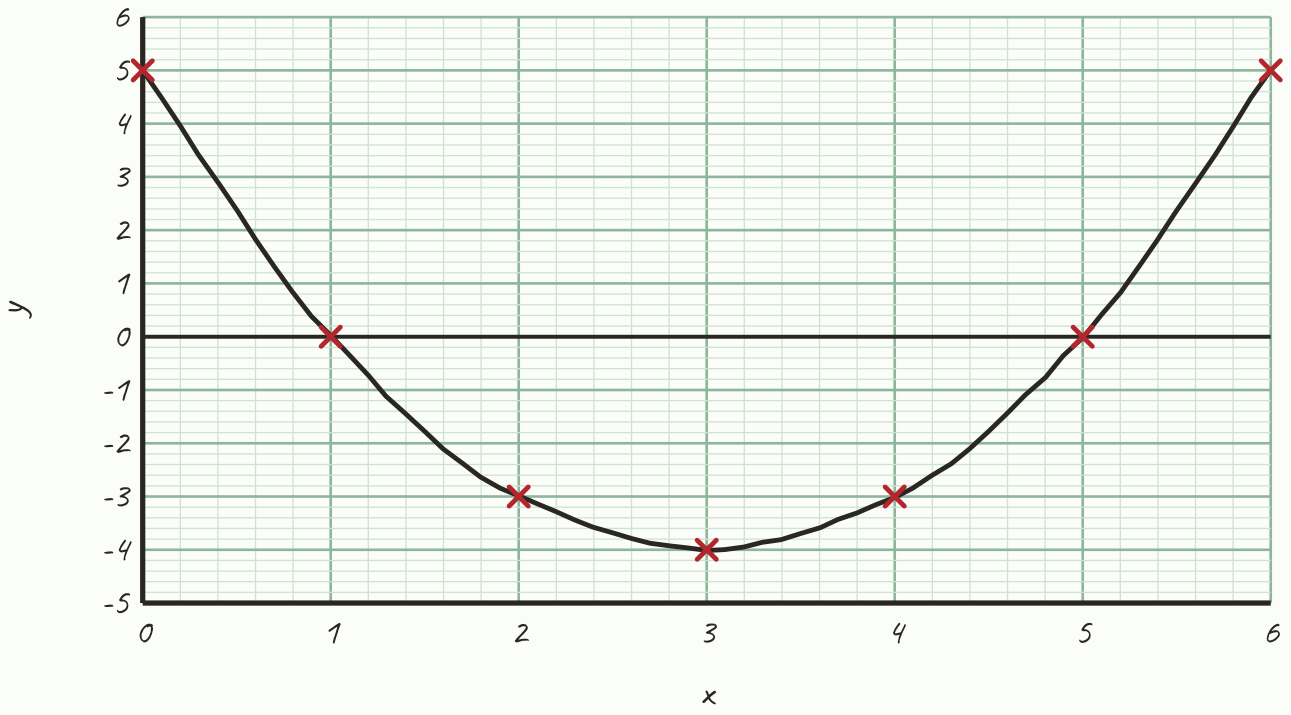
C. The roots are 1 and 5, so by symmetry the turning point's x-coordinate is halfway between them: (1 + 5) ÷ 2 = 3. Completing the square gives $(x - 3)^2 - 4$, so the minimum value is -4. A positive x^2 coefficient always gives a minimum, not a maximum.

Part two · Algebra

5 Quadratic and other graphs

Use it

$y = x^2 - 6x + 5$: roots, turning point and the symmetry between them



HT Find the roots and the turning point of $y = x^2 - 6x + 5$, showing your method. [4 marks]

Factorise: $(x - 1)(x - 5) = 0$, so $x = 1$ ✓ or $x = 5$ ✓.
Completing the square: $x^2 - 6x + 5 = (x - 3)^2 - 9 + 5 = (x - 3)^2 - 4$ ✓, so the turning point is $(3, -4)$ ✓.

4/4

M1 for a correct factorisation. A1 for both roots, x = 1 and x = 5. M1 for completing the square correctly: half of -6 is -3, and $-3^2 = 9$ must be subtracted back off. A1 for the turning point (3, -4): the sign inside the bracket flips for the x-coordinate.

It could be asked as

- Sketch a quadratic, cubic, reciprocal or exponential graph, labelling key features.
- HT Find the roots and turning point of a quadratic by factorising and completing the square.
- Use a graph to find an approximate solution to an equation.
- HT Use iteration to find an approximate solution to an equation, given a recursive formula.

Cover both pages and write your sixty seconds from memory

predict ___ / 10 got ___ / 10

Quick-fire. Answers are on page 48

5.1 How many roots does a quadratic graph that touches the x-axis once have?

5.2 State the equation of the asymptotes of $y = 1/x$.

5.3 HT Complete the square for $x^2 + 4x + 1$.

5.4 What shape is the graph of $y = 2^x$?

Part two · Algebra

6 Solving equations and inequalities

Learn it

In sixty seconds

Solve a linear equation by doing the same operation to both sides until the unknown is alone. Quadratic equations can be solved by factorising, completing the square or the quadratic formula. Two simultaneous equations are solved together, and an inequality's solution is usually a range of values rather than a single number.

Facts to know

Try it, then colour the circle: ● knew it ◐ shaky ○ missed

- ○ Solve a **linear equation** in one unknown algebraically, including those with the unknown on both sides and those with brackets, by doing the same operation to both sides.
- ○ Find an approximate solution to an equation using a graph: the x-value where the line or curve crosses the x-axis, or where two graphs intersect.
- ○ Solve a **quadratic equation** algebraically by **factorising**. HT Including those that need rearranging first, and solving by **completing the square** or using the **quadratic formula** (given in the Appendix).
- ○ Solve two **simultaneous equations** in two variables (linear/linear) algebraically, by elimination or substitution, and check the solution works in both equations.
- ○ HT Solve a simultaneous pair of equations where one is linear and one is **quadratic**, usually by substitution.
- ○ Solve a linear **inequality** in one variable, and represent the solution set on a number line, using an open circle for a strict inequality (<, >) and a closed circle for ≤ or ≥.
- ○ HT Solve a linear inequality in **two variables**, or a **quadratic inequality** in one variable, and represent the solution set using set notation and on a graph (a dashed line for a strict inequality, a solid line for ≤ or ≥).
- ○ Translate a situation or a written procedure into an algebraic expression or formula, and derive an equation (or two simultaneous equations) to solve a problem, interpreting the solution in context.
- ○ HT Find an approximate solution to an equation numerically, using **iteration** with a given recursive formula and suffix notation, such as $x_{n+1} = f(x_n)$.
- ○ Know that solving an equation means finding the value or values of the unknown that make it true, and always check a solution by substituting it back into the original equation.

Key words

- **Solve** find the value (or values) of the unknown that make an equation true. ☐
- **Simultaneous equations** two or more equations that are true for the same values of the unknowns at once. ☐
- **Elimination** a method for solving simultaneous equations by adding or subtracting them to remove one unknown. ☐
- **Substitution** replacing a letter with its value, or with an expression for it, from another equation. ☐
- **Inequality** a statement that one expression is greater than, or less than, another. ☐
- **Solution set** every value that satisfies an inequality, not just one number. ☐
- **Quadratic formula** the formula $x = (-b \pm \sqrt{(b^2 - 4ac)}) \div 2a$, used to solve $ax^2 + bx + c = 0$. ☐
- **Iteration** repeating a calculation, using each output as the next input, to close in on a solution. ☐

Elimination, step by step. To solve $2x + y = 11$ and $x - y = 1$ simultaneously: the y-terms have opposite signs, so add the two equations to eliminate y: $3x = 12$, so $x = 4$. Substitute back into either original equation: $4 - y = 1$, so $y = 3$. Always check both original equations with the final answer, not just one.

A 2023 Paper 1 simultaneous equations question was solved well by about half of students: the most common place to go wrong was at the subtraction or addition stage, subtracting when the matching coefficients had different signs (when they should have added) and adding when they had the same sign (when they should have subtracted). Look at the signs in front of the terms you are eliminating, not just their size, before deciding whether to add or subtract.

Spot it Which inequality is drawn?

Which inequality does an open circle at 3 with the arrow pointing right represent?

(A) $x \geq 3$ (B) $x > 3$

(C) $x < 3$ (D) $x \leq 3$

Cover this, choose, then read

B. An open circle means 3 itself is not included, which rules out ≥. The arrow points right (towards bigger numbers), which rules out the two 'less than' options. So it must be the strict inequality $x > 3$.

Part two · Algebra

6 Solving equations and inequalities

Use it

Solving 2x + y = 11 and x − y = 1 by elimination

Add the equations	Substitute back
2x + y = 11 x − y = 1 Add: 3x = 12 **x = 4**	x − y = 1 4 − y = 1 **y = 3**

Check in the *other* equation: 2(4) + 3 = 11. Correct.

Solve the simultaneous equations 2x + y = 11 and x − y = 1. [3 marks]

Add the two equations to eliminate y: 3x = 12 ✓, so x = 4 ✓. Substitute into x − y = 1: 4 − y = 1, so y = 3 ✓.

3/3

M1 for a correct method to eliminate one variable (here, adding the equations since the y-terms are +y and -y). A1 for x = 4. A1 for y = 3, found by substituting back and solving. A check in the other original equation isn't a mark on its own, but it catches an arithmetic slip before you move on.

It could be asked as

- Solve a linear equation, including one with brackets or the unknown on both sides.
- HT Solve a quadratic equation by completing the square or using the quadratic formula.
- Solve a pair of simultaneous linear equations by elimination or substitution.
- HT Solve an inequality in two variables, or a simultaneous linear/quadratic pair.

Cover both pages and write your sixty seconds from memory

predict ___ / 10 got ___ / 10

Quick-fire. Answers are on page 48

6.1 Solve 3x − 4 = 11.

6.2 Solve 5x + 2 = 2x + 14.

6.3 HT Solve $x^2 + x - 12 = 0$ by factorising.

6.4 Write the solution set of 2x + 1 ≥ 9 on a number line.

Part two · Algebra

7 Sequences

Learn it

In sixty seconds

Generate a sequence from a term-to-term rule (how to get from one term to the next) or a position-to-term rule (how to work out any term directly from its position, n). Recognise triangular, square, cube and Fibonacci-type sequences, and find the nth term of a linear or quadratic sequence.

Facts to know

Try it, then colour the circle: ● knew it ◐ shaky ○ missed

- ○ Generate the terms of a sequence from a **term-to-term rule** (such as 'add 4 to the previous term') or a **position-to-term rule** (such as 3n + 1), including from patterns and diagrams.
- ○ Recognise and use sequences of **triangular numbers** (1, 3, 6, 10, ...), **square numbers** (1, 4, 9, 16, ...), **cube numbers** (1, 8, 27, ...) and simple **arithmetic progressions**, where the term-to-term difference is constant.
- ○ Recognise **Fibonacci-type sequences**, where each term is the sum of the two before it, **quadratic sequences**, where the second difference between terms is constant, and simple **geometric progressions** r^n, where r is a positive rational number.
- ○ HT Recognise other sequences, including a geometric progression where the common ratio r is a surd.
- ○ Deduce an expression for the **nth term** of a linear sequence: find the common difference d, then the nth term is dn + (first term − d).
- ○ HT Deduce an expression for the nth term of a **quadratic sequence**: find the constant second difference, halve it for the coefficient of n^2, then adjust to match the actual terms.
- ○ Use the nth term of a sequence to find a specific term, or to decide whether a given number appears in the sequence by solving dn + c = (that number) for n and checking it's a positive integer.
- ○ A sequence can go up (increasing), down (decreasing), or follow a repeating pattern; read the question carefully to see which is being described.
- ○ Sequences can model real situations, such as the number of tiles needed to surround a growing pattern, or a pattern of matchsticks.
- ○ Identify the type of sequence from its first few terms by checking the differences between consecutive terms: a constant first difference is linear, a constant second difference is quadratic.

Key words

- **Term** one number in a sequence. ☐
- **Term-to-term rule** a rule describing how to get from one term to the next. ☐
- **Position-to-term rule** a rule (the nth term) giving any term directly from its position, n. ☐
- **Arithmetic sequence** a sequence with a constant difference between consecutive terms. ☐
- **Geometric progression** a sequence where each term is found by multiplying the previous one by a constant ratio. ☐
- **Common difference** the constant amount added (or subtracted) between consecutive terms of an arithmetic sequence. ☐
- **Second difference** the difference between consecutive first differences; constant for a quadratic sequence. ☐
- **Fibonacci-type sequence** a sequence where each term is the sum of the two terms before it. ☐

Finding the nth term of a quadratic sequence. For 3, 8, 15, 24, 35: the first differences are 5, 7, 9, 11, and the second difference is a constant 2. Halve the second difference to get the n^2 coefficient: n^2. Compare n^2 (1, 4, 9, 16, 25) with the sequence (3, 8, 15, 24, 35): each term is 2n bigger, so the nth term is $n^2 + 2n$.

A 2023 Paper 2 question gave four terms of a quadratic sequence and asked for the nth term: many students scored one mark for correctly finding the second differences, which was by far the most common successful first step, but a significant number did not know how to proceed from there. After halving the second difference, subtract that quadratic part from the actual terms to see what linear or constant part is left over, rather than guessing.

Spot it Which type of sequence?

7, 11, 15, 19, 23, ... is an example of which type of sequence?

(A) Geometric, with common ratio 4

(B) Quadratic, with a constant second difference of 4

(C) A Fibonacci-type sequence, built from the two terms before

(D) Arithmetic, common difference 4

Cover this, choose, then read

D. The difference between consecutive terms is always 4 (a constant first difference), which is exactly what defines an arithmetic sequence. A geometric sequence would multiply by 4 each time (7, 28, 112, ...), and a quadratic sequence needs a constant second difference, not first.

Part two · Algebra

7 Sequences

Use it

From term-to-term differences to the nth term

Term	n = 1	n = 2	n = 3	n = 4	n = 5
Sequence	3	8	15	24	35
1st difference		5	7	9	11
2nd difference			2	2	2

A constant second difference of 2 means the nth term starts with n^2. Comparing n^2 (1, 4, 9, 16, 25) to the sequence shows each term is 2n more, so the nth term is $n^2 + 2n$.

HT Find an expression for the nth term of the quadratic sequence 3, 8, 15, 24, 35, ... [3 marks]

First differences: 5, 7, 9, 11 ✓. Second difference is constant at 2, so the n^2 coefficient is 2 ÷ 2 = 1 ✓.
Comparing n^2 = 1, 4, 9, 16, 25 with the sequence shows each term is 2n bigger, so the nth term is $n^2 + 2n$ ✓.

3/3

M1 for finding the first and second differences. M1 for halving the second difference to get the coefficient of n^2 (here, 1, so the first term is n^2). A1 for the complete expression, $n^2 + 2n$, found by comparing n^2 to the sequence and spotting the linear part left over.

It could be asked as

- Find the next term, or a given term, of a sequence from a term-to-term or position-to-term rule.
- Find the nth term of a linear sequence, and use it to find a specific term or test if a number is in the sequence.
- HT Find the nth term of a quadratic sequence.
- Recognise a triangular, square, cube, Fibonacci-type or geometric sequence from its terms.

Cover both pages and write your sixty seconds from memory

predict ___ / 10 got ___ / 10

Quick-fire. Answers are on page 48

7.1 Find the next two terms of 2, 6, 10, 14, ...

7.2 Find the nth term of 5, 8, 11, 14, ...

7.3 Is 50 a term in the sequence with nth term 4n + 1?

7.4 HT Find the nth term of 2, 5, 10, 17, 26, ...

Part three · Ratio, proportion and rates of change

8 Ratio and proportion

Learn it

In sixty seconds

A ratio compares two or more quantities. Divide an amount in a given ratio by splitting it into equal parts. Direct proportion means two quantities increase together at a constant rate; inverse proportion means one increases as the other decreases, so their product stays constant. Use unit pricing to compare value for money.

Facts to know

Try it, then colour the circle: ● *knew it* ◐ *shaky* ○ *missed*

- ○ Use **ratio notation**, including reducing a ratio to its simplest form (12 : 18 = 2 : 3), by dividing both parts by their highest common factor.
- ○ Divide a quantity into two (or more) parts in a given **part : part** ratio, or a **part : whole** ratio, by finding the total number of parts first.
- ○ Express one quantity as a **fraction** of another, where the fraction can be less than 1 or greater than 1, and express a multiplicative relationship between two quantities as a ratio or a fraction.
- ○ Apply ratio to real contexts: conversion, comparison, scaling, mixing and concentrations, including **better value** or **best-buy** problems, by comparing the price per common unit (such as price per 100 g).
- ○ Use scale factors, scale diagrams and maps, including geometrical problems: a scale of 1 : 20000 means 1 cm on the map represents 20000 cm (200 m) in real life.
- ○ Understand and use **proportion** as equality of ratios: if a : b = c : d, the ratios a/b and c/d are equal, and relate ratios to fractions and to linear functions.
- ○ Solve problems involving **direct proportion** (both quantities increase or decrease together at a constant rate, y = kx) and **inverse proportion** (as one increases the other decreases, so xy = k is constant), including graphical and algebraic representations.
- ○ Recognise that 'X is inversely proportional to Y' is equivalent to 'X is proportional to 1/Y', and interpret an equation that describes direct or inverse proportion.
- ○ HT **Construct** an equation that describes a direct or inverse proportion relationship from given information (not just interpret one that's already given), including finding the constant of proportionality k.
- ○ Identify and work with fractions in ratio problems, such as converting a ratio to a fraction of the total.

Key words

- **Ratio** a way of comparing two or more quantities, written as a : b. ☐
- **Direct proportion** two quantities that increase or decrease together at a constant rate, y = kx. ☐
- **Inverse proportion** one quantity increases as the other decreases, so their product is constant, y = k/x. ☐
- **Constant of proportionality** the fixed number k that links two proportional quantities. ☐
- **Unit price** the cost of one unit of a product, such as price per 100 g, used to compare value. ☐
- **Scale factor** the number that multiplies a length in one shape or map to give the matching length in another. ☐
- **Part : whole ratio** a ratio comparing one part to the total amount, rather than to another part. ☐
- **Simplest form** a ratio written with the smallest possible whole numbers, found by dividing by the HCF. ☐

Why you divide by the total number of parts. To share £72 in the ratio 5 : 3, there are 5 + 3 = 8 equal parts in total. One part is £72 ÷ 8 = £9. The first share is 5 × £9 = £45, and the second is 3 × £9 = £27. Check: £45 + £27 = £72, and 45 : 27 simplifies back to 5 : 3.

A 2023 Paper 1 ratio-sharing question was answered well overall, but the most common error was to take only one part of the ratio from the total instead of finding the value of one share first: working out the total number of parts before dividing is the step that goes missing most often. Another classic slip, seen across several series, is adding the two sides of the ratio when the question actually asked for a comparison or a difference.

Spot it Which share is bigger?

£108 is shared in the ratio 7 : 5. What is the larger share?

(A) £63 (B) £54

(C) £45 (D) £72

Cover this, choose, then read

A. There are 7 + 5 = 12 parts, so one part is £108 ÷ 12 = £9. The larger share is 7 × £9 = £63. £54 comes from sharing equally instead of using the ratio, and the others use the wrong number of parts.

Part three · Ratio, proportion and rates of change

8 Ratio and proportion

Use it

Two tubs of the same yoghurt: which is better value?

Tub A: 800 g for £2.80	Tub B: 500 g for £1.95
£2.80 ÷ 800 × 100 = £0.35 per 100 g	£1.95 ÷ 500 × 100 = £0.39 per 100 g

Tub A costs less per 100 g (35p against 39p), so Tub A is the better buy, even though it costs more overall.

Tub A is 800 g for £2.80. Tub B is 500 g for £1.95. Which is better value for money? Show your working. [3 marks]

Tub A: £2.80 ÷ 800 × 100 = £0.35 per 100 g ✓. Tub B: £1.95 ÷ 500 × 100 = £0.39 per 100 g ✓. Tub A is cheaper per 100 g, so Tub A is better value ✓.

3/3

M1 for a correct unit-price method for one tub (price ÷ size, scaled to a common unit). A1 for both correct unit prices, 35p and 39p. A1 for the conclusion: it must refer back to the two prices and name the better tub, not just state a number.

It could be asked as

- Divide a quantity into a given ratio, or compare ratios.
- Solve a best-buy or better-value problem using unit pricing.
- Solve a direct or inverse proportion problem, including scaling a recipe or a mixture.
- HT Construct an equation linking two directly or inversely proportional quantities, and find the constant of proportionality.

Cover both pages and write your sixty seconds from memory

predict ___ / 10 got ___ / 10

Quick-fire. Answers are on page 48

8.1 Simplify the ratio 18 : 24.

8.2 Share £50 in the ratio 2 : 3.

8.3 y is directly proportional to x. When x = 4, y = 20. Find y when x = 7.

8.4 HT y is inversely proportional to x. When x = 2, y = 12. Find y when x = 3.

Part three · Ratio, proportion and rates of change

9 Percentages and financial maths

Learn it

In sixty seconds

A percentage is a number of parts per hundred. Use a multiplier to increase or decrease by a percentage in one step. Compound interest adds interest onto a growing total each year, using the formula in the Appendix. Work backwards from a final amount to an original value by dividing by the multiplier, not by the percentage.

Facts to know

Try it, then colour the circle: ● *knew it* ◐ *shaky* ○ *missed*

- ○ Define percentage as 'number of parts per hundred'; interpret a percentage, or a percentage change, as a fraction or a decimal, used multiplicatively (15% is ×0.15, or a 15% increase is ×1.15).
- ○ Express one quantity as a percentage of another, and compare two quantities using percentages.
- ○ Work with percentages greater than 100%: a 150% increase means the new amount is 2.5 times the original (1 + 1.5).
- ○ Solve **percentage change** problems: percentage increase or decrease, found using (change ÷ original) × 100, and **original value** problems, found by dividing the final amount by the multiplier, not the percentage.
- ○ Calculate **simple interest**, where the same amount of interest is added each period, in financial contexts.
- ○ Set up, solve and interpret **growth and decay** problems, including **compound interest**, using the formula total = $P(1 + r/100)^n$ from the Appendix, where P is the amount invested, r is the percentage rate and n is the number of time periods.
- ○ HT Work with **general iterative processes** for growth and decay, where the rate or the amount changes at each step rather than staying constant.
- ○ Know the vocabulary of household finance: profit, loss, cost price, selling price, debit, credit, balance, income tax, VAT and interest rate.
- ○ Apply the four operations to decimals and percentages in financial contexts, including questions that mix VAT, discounts and multiple percentage changes in the same problem.
- ○ Recognise that applying a percentage increase then the same percentage decrease does not return you to the original value, because the second change acts on a different amount.

Key words

- **Multiplier** the single decimal you multiply by to apply a percentage change, such as 1.15 for a 15% increase. ☐
- **Percentage change** the increase or decrease in a quantity, expressed as a percentage of the original. ☐
- **Original value** the amount before a percentage change was applied. ☐
- **Simple interest** interest calculated only on the original amount, the same each period. ☐
- **Compound interest** interest calculated on the running total, so it grows each period. ☐
- **Depreciation** a decrease in value over time, often by a constant percentage each period. ☐
- **VAT** Value Added Tax, a percentage added to the price of most goods and services. ☐
- **Principal** the original sum of money invested or borrowed, before interest. ☐

Why original-value problems divide, not subtract a percentage. If a price of £276 already includes a 15% increase, the £276 represents 115% of the original, not 100% minus 15%. Divide by the multiplier: £276 ÷ 1.15 = £240. Subtracting 15% of £276 instead would wrongly take 15% of the wrong total.

A 2023 Paper 3 compound growth question found many students using a simple interest method by mistake, multiplying one year's amount by the number of years instead of compounding it, and stating '4% of …' was not accepted as a method on its own without the actual working shown. A year-by-year calculation is valid but prone to rounding errors building up: using the single power formula from the Appendix, $P(1 + r/100)^n$, is faster and avoids that risk entirely.

Spot it Which calculation finds the original price?

A coat now costs £84 after a 20% increase. Which calculation finds the original price?

(A) £84 × 0.8
(B) £84 − £16.80
(C) £84 ÷ 1.2
(D) £84 ÷ 0.8

Cover this, choose, then read

C. £84 represents 120% of the original price (100% + 20%), so divide by the multiplier 1.2 to get back to 100%. The other options either apply a decrease instead of reversing an increase, or divide by the wrong multiplier.

Part three · Ratio, proportion and rates of change

9 Percentages and financial maths

Use it

£3200 invested at 2.5% compound interest for 3 years

End of year	Calculation	Total
1	3200 × 1.025	£3280.00
2	3280.00 × 1.025	£3362.00
3	3362.00 × 1.025	£3446.05

The quick way: $3200 \times 1.025^3 = £3446.05$ in one calculation, rather than three separate steps.

£3200 is invested for 3 years at 2.5% per year compound interest. Work out the total value of the investment, to the nearest penny. [3 marks]

Multiplier = 1 + 2.5 ÷ 100 = 1.025 ✓. Total = 3200×1.025^3 ✓ = £3446.05 ✓.

3/3

M1 for finding the correct multiplier, 1.025. M1 for using the multiplier raised to the power 3 (or three repeated multiplications), applied to £3200. A1 for the final answer, £3446.05, correctly rounded to the nearest penny.

It could be asked as

- Increase or decrease an amount by a percentage using a multiplier.
- Find the original value before a percentage increase or decrease.
- Solve a compound interest or depreciation problem using the formula in the Appendix.
- HT Work with a growth or decay problem where the rate changes at each step.

Cover both pages and write your sixty seconds from memory

predict ___ / 10 got ___ / 10

Quick-fire. Answers are on page 48

9.1 Increase £60 by 8%.

9.2 A price of £45 is after a 10% decrease. Find the original price.

9.3 Find the simple interest on £500 at 3% per year for 4 years.

9.4 £2000 grows at 5% compound interest for 2 years. Find the total.

Part three · Ratio, proportion and rates of change

10 Compound measures and rates of change

Learn it

In sixty seconds

A compound measure combines two other measures, such as speed (distance ÷ time), density (mass ÷ volume) and pressure (force ÷ area). The gradient of a straight-line graph gives a constant rate of change; on a curve, the gradient changes, and finding it at one point needs a tangent.

Facts to know

Try it, then colour the circle: ● *knew it* ◐ *shaky* ○ *missed*

- ○ Change freely between related standard units (time, length, area, volume or capacity, mass) and compound units (speed, rates of pay, prices), including in numerical contexts.
- ○ Use the compound measure **speed** = distance ÷ time, and rearrange it to find distance (speed × time) or time (distance ÷ speed); always convert units (such as minutes to hours) consistently first.
- ○ Use the compound measure **density** = mass ÷ volume, usually in kg/m^3 or g/cm^3, and **pressure** = force ÷ area, usually in N/m^2 (pascals), including rearranging either formula and comparing two compound measures.
- ○ Use compound units such as speed, rates of pay and unit pricing, including making comparisons between two or more options.
- ○ Interpret the gradient of a straight-line graph, such as a distance-time graph, as a rate of change: a steeper gradient means a faster rate.
- ○ Recognise and interpret graphs that illustrate direct and inverse proportion, including the characteristic curve of an inverse proportion graph.
- ○ HT Interpret the gradient **at a point** on a curve as the **instantaneous rate of change** at that exact point, found by drawing a tangent to the curve and finding its gradient.
- ○ HT Apply the concepts of **average** rate of change (the gradient of a chord, joining two points on a curve) and **instantaneous** rate of change (the gradient of a tangent, at a single point), in numerical, algebraic and graphical contexts.
- ○ A distance-time graph's gradient gives speed; a velocity-time graph's gradient gives acceleration, and the area underneath gives distance travelled (see page 12 for gradients and areas of curved graphs).
- ○ Use standard compound measures in algebraic contexts too, such as writing an expression for the time taken in terms of a variable speed or distance.

Key words

- **Compound measure** a measure made by combining two other measures, such as speed or density. ☐
- **Density** mass per unit volume of a substance: density = mass ÷ volume. ☐
- **Pressure** force per unit area: pressure = force ÷ area. ☐
- **Average speed** total distance travelled divided by total time taken. ☐
- **Rate of change** how quickly one quantity changes as another quantity changes. ☐
- **Chord** a straight line joining two points on a curve. ☐
- **Tangent to a curve** a straight line that touches a curve at one point and has the same gradient there. ☐
- **Instantaneous rate** the rate of change at one exact moment or point, not averaged over an interval. ☐

Why units must match before you divide. Speed = distance ÷ time only gives a sensible answer if the units are consistent: 126 km in 1 hour 48 minutes needs the time converted to 1.8 hours (48 minutes = 0.8 of an hour) before dividing, giving 126 ÷ 1.8 = 70 km/h. Using 1.48 hours by mistake is a very common error, since 48 minutes is not 0.48 of an hour.

Speed, distance and time questions are consistently flagged by examiners as a weak area: a 2023 Paper 3 question needed a time given in hours and minutes converted correctly, and common wrong conversions included rounding 3 hours 40 minutes to 3.6, 3.7 or 4 hours instead of keeping the exact value, 3 + 40/60 = 3.67 hours. Convert minutes to a fraction (or decimal) of an hour by dividing by 60, never by just moving the decimal point.

Spot it Which has the greater density?

Block A has mass 540 g and volume 200 cm^3. Block B has mass 480 g and volume 150 cm^3. Which has the greater density?

(A) Block A (2.7 g/cm^3 against 2.4 g/cm^3)

(B) Block B (3.2 g/cm^3 against 2.7 g/cm^3)

(C) They have equal density

(D) Block A, because it has the greater mass

Cover this, choose, then read

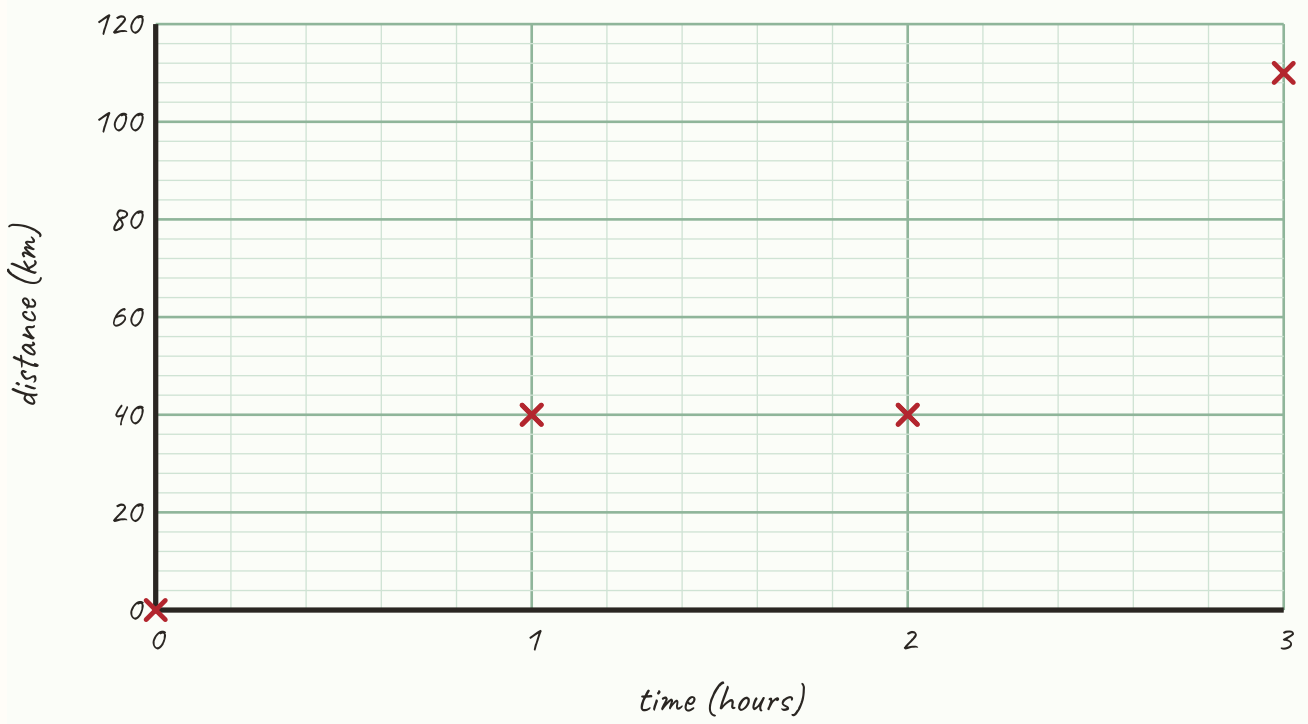
B. Density = mass ÷ volume. Block A: 540 ÷ 200 = 2.7 g/cm^3. Block B: 480 ÷ 150 = 3.2 g/cm^3. Block B is denser, even though it has less mass, because its volume is smaller in proportion.

Part three · Ratio, proportion and rates of change

10 Compound measures and rates of change

Use it

A distance-time graph: constant speed, then stationary, then faster



The flat section (hour 1 to 2) has gradient zero: stationary. The steepest section (hour 2 to 3) is the fastest speed, 70 km/h.

A car travels 126 km in 1 hour 48 minutes. Work out its average speed in km/h. [2 marks]

48 minutes = 48 ÷ 60 = 0.8 hours ✓, so the time is 1.8 hours. Speed = 126 ÷ 1.8 = 70 km/h ✓.

2/2

M1 for converting 48 minutes to 0.8 hours (dividing by 60, not by 100). A1 for the correct final speed, 70 km/h, from distance ÷ total time in hours.

It could be asked as

- Calculate speed, density or pressure, or rearrange the formula to find a different quantity.
- Convert between units of time (minutes and hours) correctly within a compound-measure question.
- Interpret the gradient of a distance-time or velocity-time graph.
- HT Find or interpret the gradient at a point on a curve, or compare an average rate of change with an instantaneous one.

Cover both pages and write your sixty seconds from memory

predict ___ / 10 got ___ / 10

Quick-fire. Answers are on page 48

10.1 A journey of 150 miles takes 3 hours. Find the average speed.

10.2 Find the density of a block with mass 90 g and volume 36 cm^3.

10.3 A force of 450 N acts over an area of 0.03 m^2. Find the pressure.

10.4 HT What does the gradient of a tangent to a curve at a point represent?

Part four · Geometry and measures

11 Angles, polygons, constructions, bearings and loci

Learn it

In sixty seconds

Angles at a point sum to 360°, and angles on a straight line sum to 180°. Parallel lines cutting a transversal give equal alternate and corresponding angles. Use a ruler and compasses for standard constructions, and give a bearing as a three-figure angle measured clockwise from north.

Facts to know

Try it, then colour the circle: ● knew it ◐ shaky ○ missed

- ○ Angles at a **point** sum to 360°. Angles at a point on a **straight line** sum to 180°. **Vertically opposite** angles, formed where two lines cross, are equal.
- ○ On **parallel lines** cut by a transversal: **alternate angles** ('Z', informally) are equal, and **corresponding angles** ('F', informally) are equal. Never use the informal shape names in a written answer.
- ○ The angles in a triangle sum to 180°. Use this for the angle sum of any polygon: a regular n-sided polygon has exterior angles of 360° ÷ n, and each interior angle is 180° minus the exterior angle.
- ○ Know the properties of special quadrilaterals (square, rectangle, parallelogram, trapezium, kite, rhombus) and triangles (isosceles, equilateral, scalene, right-angled, acute, obtuse), and the names pentagon, hexagon, octagon, decagon.
- ○ Use the standard ruler-and-compass **constructions**: the perpendicular bisector of a line segment, a perpendicular to a line from or at a point, and bisecting an angle. The perpendicular distance from a point to a line is the shortest distance to it.
- ○ Use these constructions to build a given figure, or solve a **loci** problem: a locus is every point fitting a rule, such as 'equidistant from two points' (perpendicular bisector) or 'equidistant from two lines' (angle bisector).
- ○ Measure line segments and angles, including maps and scale drawings, and use **bearings**: the eight compass points, and three-figure bearings (000° to 360°, always clockwise from north, written with three digits such as 045°).
- ○ Use the basic congruence criteria for triangles: SSS (three sides), SAS (two sides and the included angle), ASA (two angles and a side), and RHS (right angle, hypotenuse, side).
- ○ Draw an accurate diagram from a written description, using the conventions for labelling points, lines, vertices and the sides and angles of a triangle.
- ○ Identify and describe the symmetries (reflection and/or rotation) of a 2D shape.

Key words

- **Vertically opposite angles** equal angles formed on opposite sides of where two straight lines cross. ☐
- **Alternate angles** equal angles on opposite sides of a transversal crossing two parallel lines. ☐
- **Corresponding angles** equal angles in matching positions where a transversal crosses two parallel lines. ☐
- **Transversal** a line that crosses two or more other lines. ☐
- **Locus** the set of all points that satisfy a given rule or condition (plural: loci). ☐
- **Bearing** a direction given as an angle, in three figures, measured clockwise from north. ☐
- **Bisector** a line that divides a line segment or an angle into two exactly equal parts. ☐
- **Congruent** identical in shape and size; one shape would fit exactly on top of the other. ☐

Why a regular polygon's exterior angles sum to 360°. Imagine walking around the edge of any polygon, turning through each exterior angle at every corner: by the time you're back where you started, facing the way you began, you've turned through one full turn, 360°. Divide 360° by the number of sides for a regular polygon's exterior angle, then subtract from 180° for the interior angle.

A 2023 question gave angles on a straight line in terms of x: almost every student knew they summed to 180°, but some divided by the wrong number (the number of terms, not the total of the coefficients), and some solved for x correctly but then forgot to multiply back up for the angle actually asked for.

Spot it Which angle fact applies?

Two parallel lines are cut by a transversal. One angle is 72°. Which angle must also be 72°, without more information?

(A) Every other angle at the same intersection

(B) Its co-interior angle on the same side

(C) The angle on the other parallel line that is not in the matching position

(D) Its vertically opposite angle, or its corresponding angle

Cover this, choose, then read

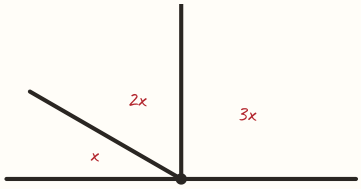
D. Vertically opposite angles are always equal, and a corresponding angle (same relative position at the other intersection) is also always equal. Co-interior angles on parallel lines add to 180°, not equal; the other angles at the same point are generally different unless more information is given.

Part four · Geometry and measures

11 Angles, polygons, constructions, bearings and loci

Use it

Angles on a straight line: x, 2x and 3x



x + 2x + 3x = 180°, so 6x = 180° and x = 30°. The three angles are 30°, 60° and 90°.

Three angles on a straight line are x, 2x and 3x. Work out the size of the largest angle. [3 marks]

x + 2x + 3x = 180 ✓ (angles on a straight line). 6x = 180, so x = 30 ✓. The largest angle is 3x = 3 × 30 = 90° ✓.

3/3

M1 for setting up the equation using the correct angle fact (angles on a straight line sum to 180°). A1 for solving to find x = 30. A1 for identifying and calculating the largest angle, 90°, not just stating x: read exactly what the question asks for.

It could be asked as

- Find a missing angle using angle facts at a point, on a line, or between parallel lines.
- Work out the interior or exterior angle of a regular polygon.
- Construct a perpendicular bisector, an angle bisector, or solve a loci problem.
- Give or interpret a three-figure bearing from a diagram or a scale drawing.

Cover both pages and write your sixty seconds from memory

predict ___ / 10 got ___ / 10

Quick-fire. Answers are on page 48

11.1 Two angles on a straight line are 2x and 3x. Find x.

11.2 Find the exterior angle of a regular 9-sided polygon.

11.3 Find the interior angle of a regular 9-sided polygon.

11.4 What construction gives every point equidistant from two given points?

Part four · Geometry and measures

12 Perimeter, area and volume

Learn it

In sixty seconds

Learn the area formulae for triangles, parallelograms, trapezia and circles, and the volume formula for a prism: area of cross-section times length. Surface area and volume of spheres, cones and pyramids follow from formulae in the Appendix; some are given on the exam paper and some are not.

Facts to know

Try it, then colour the circle: ● knew it ◐ shaky ○ missed

- ○ Know and apply the formulae to calculate the **area** of a triangle (½ × base × height), a parallelogram (base × height) and a **trapezium** (½(a + b)h, where a and b are the parallel sides and h is the perpendicular gap between them).
- ○ Calculate the **perimeter** of a 2D shape, including a circle, and the area of a circle and composite shapes made of more than one part.
- ○ Know the circle formulae: **circumference** = $2\pi r = \pi d$, and **area** = πr^2. These must be recalled: they are not given on the exam paper.
- ○ Calculate the **volume of a prism** (including a cylinder) using area of cross-section × length; a prism has the same cross-section all the way through.
- ○ Calculate the **surface area and volume of a sphere, a pyramid, a cone** and composite solids built from them, including a frustum (a cone with the top sliced off). The sphere and cone formulae are given on the exam paper; the prism and trapezium formulae are not.
- ○ Calculate **arc lengths**, sector angles and the **area of a sector** of a circle, by treating the sector as a fraction of the whole circle: (angle ÷ 360) × circumference, or (angle ÷ 360) × area.
- ○ Identify the properties of the faces, surfaces, edges and vertices of cubes, cuboids, prisms, cylinders, pyramids, cones and spheres.
- ○ Interpret and construct plans and elevations of 3D shapes: the views looking from directly above, from the front, and from the side.
- ○ Use standard units of area (cm^2, m^2) and volume (cm^3, m^3), and convert between them using the correct squared or cubed conversion factor (1 m^2 = 10000 cm^2, 1 m^3 = 1000000 cm^3).
- ○ Solutions may be left in terms of π, which is exact, rather than rounded to a decimal, unless the question asks for a decimal answer.

Key words

- **Trapezium** a quadrilateral with exactly one pair of parallel sides. ☐
- **Cross-section** the 2D shape you'd see if you sliced straight through a solid. ☐
- **Prism** a 3D shape with a constant cross-section along its length. ☐
- **Frustum** a cone (or pyramid) with the top sliced off by a cut parallel to the base. ☐
- **Arc** part of the circumference of a circle. ☐
- **Sector** a 'pie slice' region of a circle, bounded by two radii and an arc. ☐
- **Composite shape** a shape made by combining two or more simpler shapes. ☐
- **Surface area** the total area of every face or curved surface of a 3D shape. ☐

Which formulae you must know, and which are given. The Appendix (pages 38-39) splits formulae into three groups: ones you must memorise and are never given (the circle formulae, the trapezium area, prism volume), ones you must know or derive (compound interest, the probability rules), and ones that genuinely are printed on the exam paper (sphere and cone surface area and volume). Knowing which is which saves revision time.

A 2023 question comparing the surface areas of a hemisphere and a cylinder by ratio was poorly answered: many students omitted a flat circular face entirely, and an equal proportion worked out a volume instead of the surface area that was actually asked for. Before calculating anything, sketch the solid and tick off every separate face or surface you can see, including any flat faces created by slicing a shape, so none gets forgotten.

Spot it — Which calculation finds the volume?

A cylinder has radius 4 cm and height 10 cm. Which calculation gives its volume?

(A) $\pi \times 4^2 \times 10$ (B) $2 \times \pi \times 4 \times 10$

(C) $\pi \times 4 \times 10$ (D) $\pi \times 4^2 + 10$

Cover this, choose, then read

A. Volume of a cylinder is area of the circular cross-section (πr^2) times the length (height): $\pi \times 4^2 \times 10$. The other options find a curved surface area, muddle the formula, or add instead of multiply.

Part four · Geometry and measures

12 Perimeter, area and volume

Use it

A trapezium and a cylinder: area and volume

Trapezium: a = 8, b = 14, h = 6	**Cylinder: r = 4, h = 10**
Area = ½ × (8 + 14) × 6 **= 66**	Volume = π × 4^2 × 10 **= 502.7 (1 d.p.)**

A sphere has radius 6 cm. Work out its volume and its surface area, giving your answers to 1 decimal place. (Volume = $\frac{4}{3}\pi r^3$, surface area = $4\pi r^2$, both given on the formulae sheet.) [4 marks]

Volume = $\frac{4}{3}$ × π × 6^3 = 904.8 cm^3 ✓ (1 d.p.). Surface area = 4 × π × 6^2 = 452.4 cm^2 ✓ (1 d.p.).

4/4

M1 for substituting r = 6 correctly into the volume formula. A1 for the volume, 904.8 cm^3. M1 for substituting into the surface area formula. A1 for the surface area, 452.4 cm^2. Both formulae are on the sheet, so the method mark is for correct substitution, not for recalling the formula from memory.

It could be asked as

- Find the area of a triangle, parallelogram, trapezium, circle or a composite 2D shape.
- Find the volume of a prism, cylinder, cone, pyramid or sphere.
- Find the surface area of a 3D shape, including a composite solid.
- Find the arc length or area of a sector of a circle.

Cover both pages and write your sixty seconds from memory

predict ___ / 10 got ___ / 10

Quick-fire. Answers are on page 48

12.1 Find the area of a trapezium with parallel sides 5 cm and 9 cm, height 4 cm.

12.2 Find the circumference of a circle with radius 5 cm, in terms of π.

12.3 Find the volume of a prism with cross-sectional area 18 cm^2 and length 7 cm.

12.4 Find the area of a sector with angle 90° and radius 8 cm, in terms of π.

Part four · Geometry and measures

13 Pythagoras and trigonometry

Learn it

In sixty seconds

Pythagoras' theorem, $a^2 + b^2 = c^2$, links the three sides of a right-angled triangle. The trigonometric ratios sine, cosine and tangent link a side and an angle. At Higher tier, the sine rule, cosine rule and the ½ab sinC area formula extend this to any triangle, not just right-angled ones.

Facts to know

Try it, then colour the circle: ● knew it ◐ shaky ○ missed

- ○ **Pythagoras' theorem:** in a right-angled triangle, $a^2 + b^2 = c^2$, where c is the hypotenuse (the longest side, opposite the right angle). Use it to find a missing side, and derive simple proofs from it.
- ○ The **trigonometric ratios** in a right-angled triangle: sin θ = opposite ÷ hypotenuse, cos θ = adjacent ÷ hypotenuse, tan θ = opposite ÷ adjacent. Remember them with SOH CAH TOA.
- ○ Apply Pythagoras and the trig ratios to find a missing angle or length in a **right-angled triangle** in a two-dimensional figure.
- ○ HT Apply Pythagoras and trigonometry to right-angled triangles, and, where possible, **general (non-right-angled) triangles**, in **two and three dimensions**, such as the diagonal of a cuboid.
- ○ Know the **exact values** of sin θ and cos θ for θ = 0°, 30°, 45°, 60° and 90°, and the exact value of tan θ for θ = 0°, 30°, 45° and 60°, without a calculator (for example, sin 30° = ½, cos 45° = √2/2, tan 60° = √3).
- ○ HT Know and apply the **sine rule**, a/sin A = b/sin B = c/sin C, to find an unknown length or angle in any triangle, using two angle-side pairs.
- ○ HT Know and apply the **cosine rule**, $a^2 = b^2 + c^2 - 2bc \cos A$, to find an unknown side (given two sides and the included angle) or an unknown angle (given all three sides).
- ○ HT Know and apply **Area = ½ab sin C** to find the area of any triangle when two sides and the included angle are known, without needing the perpendicular height.
- ○ Decide which approach a triangle problem needs: a right angle present means Pythagoras or SOH CAH TOA; no right angle means the sine rule, the cosine rule, or ½ab sinC, depending on what's given.
- ○ Use the standard conventions for labelling a triangle: side a is opposite angle A, side b is opposite angle B, and so on, which is essential for using the sine and cosine rules correctly.

Key words

- **Hypotenuse** the longest side of a right-angled triangle, opposite the right angle. ☐
- **Opposite** the side across from the angle being used, in a right-angled triangle. ☐
- **Adjacent** the side next to the angle being used (but not the hypotenuse). ☐
- **Sine rule** a/sin A = b/sin B = c/sin C, linking sides and angles in any triangle. ☐
- **Cosine rule** $a^2 = b^2 + c^2 - 2bc \cos A$, used when a right angle isn't present. ☐
- **Included angle** the angle between two named sides. ☐
- **Exact value** a trig value given as a fraction or surd, not a rounded decimal. ☐
- **Angle of elevation** the angle measured upward from the horizontal to a point above. ☐

Choosing between the sine rule and the cosine rule. Use the sine rule when you know a matching angle-side pair (an angle and the side opposite it) plus one more piece of information. Use the cosine rule when you know two sides and the angle between them (to find the third side), or all three sides (to find any angle). If there's a right angle, simpler right-angled trigonometry is usually faster than either.

A 2023 question involving two right-angled triangles stacked together caught many students out: a common error was to calculate a side using only one triangle's angle against the wrong side, for example working out 4 × sin 37° when the 37° angle and the side of length 4 belonged to different triangles in the diagram. Label every triangle in a diagram separately, and check which angle and which sides actually belong together, before choosing sin, cos or tan.

Spot it Which ratio is needed?

In a right-angled triangle, you know the hypotenuse and need to find the side adjacent to a given angle. Which ratio should you use?

(A) Sine (sin θ = adjacent ÷ hypotenuse)

(B) Tangent (tan θ = adjacent ÷ hypotenuse)

(C) Cosine (cos θ = adjacent ÷ hypotenuse)

(D) Pythagoras' theorem, since no angle is involved

Cover this, choose, then read

C. You know the hypotenuse and want the adjacent side, and the ratio connecting exactly those two is cosine. Sine connects opposite and hypotenuse; tangent connects opposite and adjacent; Pythagoras needs two sides already known, which isn't the case here.

Part four · Geometry and measures

13 Pythagoras and trigonometry

Use it

Right-angled: Pythagoras. Any triangle: the sine and cosine rules

a = 9, b = 12, right angle between them

$c^2 = 9^2 + 12^2 = 225$
c = 15

HT No right angle: cosine rule

b = 8, c = 10, A = 55°
$a^2 = 8^2 + 10^2 - 2(8)(10)\cos 55°$
a = 8.5 (1 d.p.)

HT Triangle ABC has angle A = 52°, angle B = 71°, and side a = 11 cm (opposite angle A). Find side b, opposite angle B. [3 marks]

a/sin A = b/sin B ✓, so 11/sin 52° = b/sin 71°. b = 11 × sin 71° ÷ sin 52° ✓ = 13.2 cm ✓ (1 d.p.).

3/3

M1 for writing the sine rule with the correct sides and angles paired up. M1 for rearranging to make b the subject before substituting. A1 for the correct answer, 13.2 cm, correctly rounded. A common way to lose the final mark is rounding sin 71° or sin 52° too early: keep the full calculator value until the last step.

It could be asked as

- Find a missing side or angle in a right-angled triangle using Pythagoras or trigonometry.
- Recall and use an exact trig value without a calculator.
- HT Use the sine rule or the cosine rule to find a side or angle in a non-right-angled triangle.
- HT Use Area = ½ab sinC to find the area of a triangle, or apply trigonometry in three dimensions.

Cover both pages and write your sixty seconds from memory

predict ___ / 10 got ___ / 10

Quick-fire. Answers are on page 48

13.1 A right-angled triangle has shorter sides 6 cm and 8 cm. Find the hypotenuse.

13.2 State the exact value of sin 30°.

13.3 HT A triangle has sides a = 7, b = 9 and included angle C = 64°. Find its area.

13.4 HT When would you use the cosine rule rather than the sine rule?

Part four · Geometry and measures

14 Transformations, congruence and similarity

Learn it

In sixty seconds

A transformation changes a shape's position, orientation or size. Reflection, rotation and translation keep a shape congruent (identical); enlargement makes a similar shape, the same proportions but a different size. For similar solids, lengths scale by a factor k, areas by k^2, and volumes by k^3.

Facts to know

Try it, then colour the circle: ● knew it ◐ shaky ○ missed

- ○ Identify, describe and construct **congruent** and **similar** shapes, including on coordinate axes, by considering rotation, reflection, translation and enlargement.
- ○ A **reflection** needs a mirror line. A **rotation** needs a centre, an angle, and a direction (clockwise or anticlockwise). A **translation** needs a vector, describing the movement right/left and up/down.
- ○ An **enlargement** needs a centre and a scale factor: every length is multiplied by the scale factor, measured from the centre of enlargement.
- ○ Enlarge a shape using a **fractional scale factor** (which makes the image smaller than the object).
- ○ HT Enlarge a shape using a **negative scale factor**: the image appears on the opposite side of the centre of enlargement, upside down and back to front relative to the object.
- ○ HT Describe the single transformation (or the overall change and any invariance) that results from a **combination** of two or more rotations, reflections and translations applied one after another.
- ○ Use the basic **congruence criteria** for triangles: SSS (three sides match), SAS (two sides and the included angle match), ASA (two angles and a corresponding side match), RHS (right angle, hypotenuse and another side match).
- ○ In similar figures, matching sides are all in the same ratio (the **linear scale factor**); matching angles are always equal.
- ○ HT In similar figures, if the linear scale factor is k, then the **area** scale factor is k^2, and the **volume** scale factor is k^3: doubling every length makes the area 4 times bigger and the volume 8 times bigger.
- ○ Use column vector notation, such as (3, -2), to describe a translation: 3 right (or left, if negative) and 2 down (or up, if positive, by convention written as the top number across, bottom number up/down).

Key words

- **Congruent** identical in shape and size. ☐
- **Similar** the same shape (matching angles equal) but a different size (matching sides in the same ratio). ☐
- **Centre of enlargement** the fixed point that lengths are measured from during an enlargement. ☐
- **Scale factor** the number every length is multiplied by during an enlargement. ☐
- **Invariant** a point or line that stays unchanged (fixed) under a transformation. ☐
- **Object** the original shape, before a transformation is applied. ☐
- **Image** the shape produced after a transformation has been applied. ☐
- **Linear scale factor** the ratio of corresponding lengths between two similar shapes. ☐

Why area scales by k^2 and volume by k^3. Imagine doubling every length of a rectangle (k = 2): both the width and the height double, so the area (width × height) multiplies by 2 × 2 = 4, which is k^2. A cuboid's volume (length × width × height) has three dimensions all scaling by k, so the volume multiplies by k × k × k = k^3.

A 2023 question asking students to describe a single enlargement saw many answers describing it as a combination of a rotation and an enlargement, which automatically scores no marks: a single transformation needs a single, correctly named transformation. Scale factors were also often given as the wrong sign or the reciprocal, and centres of enlargement were guessed rather than found using the diagram; read off two matching points and draw the line through them to locate the centre.

Spot it — Which description is complete?

A triangle is reflected in the line x = 2. Which description correctly and fully describes this transformation?

(A) A reflection
(B) A reflection in the line x = 2
(C) A reflection in the y-axis
(D) A translation 4 units to the right

Cover this, choose, then read

B. A full description of a reflection names the specific mirror line, here x = 2. 'A reflection' alone is incomplete, and x = 2 is not the y-axis (which is x = 0). Reflections and translations are also different transformations, not interchangeable.

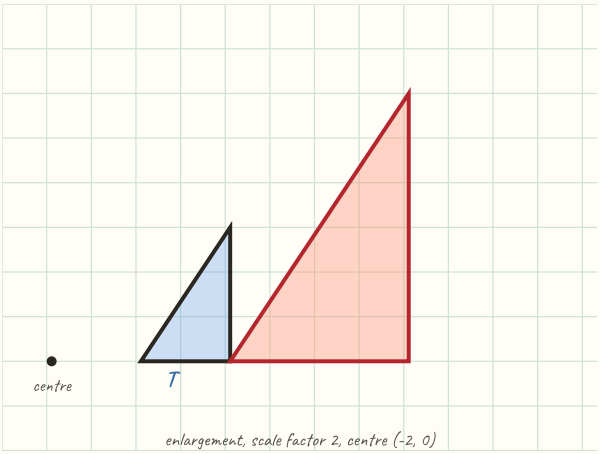
Part four · Geometry and measures

14 Transformations, congruence and similarity

Use it

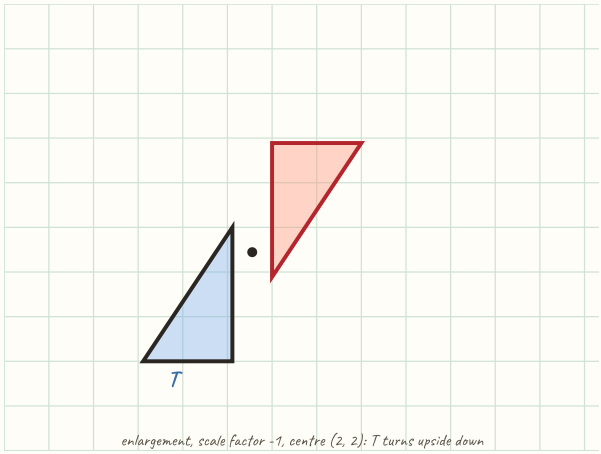
Enlargement, scale factor 2 and scale factor -1

Scale factor 2



enlargement, scale factor 2, centre (-2, 0)

HT Scale factor -1



enlargement, scale factor -1, centre (2, 2): T turns upside down

HT A model statue is 40 cm³ in volume. The real statue is similar, with every length 3 times as big. Find the volume of the real statue. [2 marks]

Volume scale factor = $3^3 = 27$ ✓. Real volume = 40 × 27 = 1080 cm³ ✓.

2/2

M1 for cubing the linear scale factor to get the volume scale factor, 27 (not using 3 or 3^2 by mistake). A1 for the final volume, 1080 cm³. The linear scale factor of 3 is never applied directly to the volume: that's the single most common way this question type loses marks.

It could be asked as

- Describe a single transformation (reflection, rotation, translation or enlargement) fully.
- HT Enlarge a shape by a negative scale factor, or describe a combination of transformations.
- Decide whether two triangles are congruent, and state which congruence criterion applies.
- HT Use the area or volume scale factor to solve a problem about similar shapes or solids.

Cover both pages and write your sixty seconds from memory

predict ___ / 10 got ___ / 10

Quick-fire. Answers are on page 48

14.1 What two pieces of information does a rotation need, besides the centre?

14.2 Two similar shapes have a linear scale factor of 4. Find the area scale factor.

14.3 HT Two similar solids have a linear scale factor of 5. Find the volume scale factor.

14.4 Which congruence criterion applies when you know two sides and the angle between them?

Part four · Geometry and measures

15 Circle theorems and vectors

Learn it

In sixty seconds

HT Every circle theorem on this page is Higher tier only. Angles in a circle follow fixed rules connecting the centre, the circumference, tangents and chords. A vector has both size and direction; add vectors nose to tail, and use them to prove geometric facts, such as showing two lines are parallel.

Facts to know

Try it, then colour the circle: ● knew it ◐ shaky ○ missed

- ○ Identify and apply circle definitions: **centre**, **radius**, **chord** (a line joining two points on the circumference), **diameter**, **circumference**, **tangent** (a line touching the circle at one point), **arc**, **sector** and **segment**.
- ○ HT The angle at the **centre** is twice the angle at the **circumference**, when both are subtended by the same arc.
- ○ HT The angle in a **semicircle** (subtended by the diameter) is always 90°.
- ○ HT Angles in the **same segment**, subtended by the same arc, are equal to each other.
- ○ HT Opposite angles in a **cyclic quadrilateral** (a quadrilateral with all four vertices on the circle) sum to 180°.
- ○ HT A **tangent** is perpendicular to the radius at the point where it touches the circle. Tangents drawn from the same external point are equal in length.
- ○ HT The **alternate segment theorem**: the angle between a tangent and a chord equals the angle in the alternate segment (the angle subtended by that chord on the other side).
- ○ HT The perpendicular from the centre of a circle to a chord **bisects the chord** (cuts it exactly in half).
- ○ Describe a translation as a **2D vector**, such as (3, -2); add and subtract vectors, and multiply a vector by a scalar (a number), both diagrammatically (nose to tail) and using column vectors.
- ○ HT Use vectors to construct a geometric argument or **proof**, such as showing two lines are parallel (one vector is a scalar multiple of the other) or finding a midpoint.

Key words

Chord a straight line joining two points on a circle's circumference.	☐
Tangent a straight line that touches a circle at exactly one point.	☐
Cyclic quadrilateral a four-sided shape with every vertex on the same circle.	☐
Subtend an angle 'subtended' by an arc or chord is the angle it forms at a given point.	☐
Vector a quantity with both magnitude (size) and direction, such as a translation.	☐
Scalar an ordinary number, used to multiply a vector and change its size (not its direction, unless negative).	☐
Parallel vectors vectors that point in the same or exactly opposite direction; one is a scalar multiple of the other.	☐
Resultant the single vector that results from adding two or more vectors together.	☐

Why the alternate segment theorem has that name. A chord drawn from the point of tangency splits the circle into two segments. The angle between the tangent and the chord equals the angle in the *other* ('alternate') segment, not the one right next to it. Spotting which segment is 'alternate' in a diagram is usually the hardest part of using this theorem correctly.

A 2022 'show that' circle theorem question needed every angle named clearly: examiners noted that stating a single angle's value on its own, without saying which circle theorem justified it or which triangle it belonged to, did not gain the mark. Some students also wrongly assumed a triangle in the diagram was isosceles or equilateral without any justification. Name the theorem you are using at each step, in words, not just the number you calculated.

Spot it Which theorem applies?

A triangle is drawn with all three vertices on a circle, and one of its sides is a diameter. What can you say about the angle opposite that diameter?

(A) It is 90°, because opposite angles in a cyclic quadrilateral sum to 180°

(B) It is 180°, because it's on a straight line

(C) It cannot be found without more information

(D) It is 90°, because the angle in a semicircle is a right angle

Cover this, choose, then read

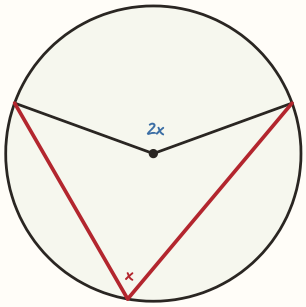
D. A diameter subtends a right angle at any point on the circumference: this is the angle-in-a-semicircle theorem, not the cyclic quadrilateral rule (which needs four vertices, not three) and not a straight-line fact, since the vertex is on the circle, not on the diameter itself.

Part four · Geometry and measures

15 Circle theorems and vectors

Use it

Angle at the centre is twice the angle at the circumference



angle at the centre is twice the angle at the circumference

If the angle at the circumference is 35°, the angle at the centre, subtended by the same arc, is 2 × 35° = 70°.

HT O is the centre of a circle. A is the midpoint of chord PQ. Prove that OA is perpendicular to PQ. [3 marks]

OP = OQ (radii) ✓. In triangles OPA, OQA: OP = OQ, PA = QA (A is the midpoint), OA is shared ✓, so the triangles are congruent (SSS). So angle OAP = angle OAQ; both lie on line PQ and are equal, so each is 90° ✓.

3/3

B1 OP = OQ as radii. M1 SSS congruence set up correctly. A1 the 90° logic completed. This proves 'the perpendicular from the centre to a chord bisects the chord' from first principles.

It could be asked as

- Find a missing angle in a circle using one or more circle theorems, explaining which theorem you used.
- Add, subtract, or find a scalar multiple of a vector, diagrammatically or using column vectors.
- Use vectors to show that two lines are parallel, or to find a midpoint or a ratio along a line.
- Combine more than one circle theorem in a single multi-step angle problem.

Cover both pages and write your sixty seconds from memory

predict ___ / 10 got ___ / 10

Quick-fire. Answers are on page 48

15.1 HT A cyclic quadrilateral has one angle of 110°. Find the opposite angle.

15.2 HT What is the angle between a tangent and the radius drawn to the point of contact?

15.3 HT If a = (2, 5) and b = (6, -1), find a + b.

15.4 HT If vector CD = 3 × vector AB, what can you conclude about the lines AB and CD?

Part five · Probability

16 Probability

Learn it

In sixty seconds

Probability is measured from 0 (impossible) to 1 (certain). For combined events, multiply probabilities along a tree diagram's branches, and add across separate routes to the same outcome. At Higher tier, conditional probability means the second event's probability depends on what happened in the first.

Facts to know

Try it, then colour the circle: ● knew it ◐ shaky ○ missed

- ○ Record, describe and analyse the frequency of outcomes of probability experiments using tables and **frequency trees**. Probabilities can be written as fractions, decimals or percentages.
- ○ Apply ideas of randomness, fairness and equally likely events to calculate the **expected** number of outcomes over a number of trials: expected number = probability × number of trials.
- ○ Relative frequency (from an experiment) gets closer to the true theoretical probability as the number of trials increases; use the probability scale from 0 to 1 with the correct vocabulary (impossible, unlikely, even chance, likely, certain).
- ○ The probabilities of an **exhaustive** set of mutually exclusive outcomes (every possible outcome, with no overlap) always sum to 1; use this to find a missing probability.
- ○ Enumerate outcomes systematically using tables, grids, **Venn diagrams** and sample space diagrams, for single and combined experiments with equally likely outcomes, and use these to calculate theoretical probabilities.
- ○ Use a **tree diagram** for combined events: multiply the probabilities **along** the branches for a single route (an 'and'), and add the probabilities of different complete routes that give the same outcome (an 'or').
- ○ Calculate the probability of independent and dependent combined events, including using tree diagrams, and know the underlying assumption: for independent events, multiply the individual probabilities; know when to add and when to multiply.
- ○ HT Calculate and interpret **conditional probability**: where the probability of a second event depends on whether a first event happened, represented using a two-way table, a tree diagram (where the second set of branches changes), or a Venn diagram.
- ○ A **Venn diagram** sorts outcomes into overlapping sets; read probabilities directly from the regions, dividing the count in a region by the total.
- ○ Decide between a tree diagram, a Venn diagram, and a two-way table based on what the question describes: overlapping categories often suit a Venn diagram, and a sequence of events suits a tree diagram.

Key words

- **Mutually exclusive** outcomes that cannot both happen at the same time. ☐
- **Exhaustive** a set of outcomes that covers every possibility, with nothing left out. ☐
- **Independent events** events where one happening does not affect the probability of the other. ☐
- **Dependent events** events where one happening changes the probability of the other. ☐
- **Conditional probability** the probability of an event, given that another event has already happened. ☐
- **Expected frequency** the number of times an outcome is predicted to happen, from probability × number of trials. ☐
- **Relative frequency** the proportion of trials in an experiment that gave a particular outcome. ☐
- **Sample space** the complete list of every possible outcome of an experiment. ☐

Without replacement changes the second probability. A bag has 4 red balls out of 10. The chance the first ball is red is 4/10. If it is not replaced, only 9 balls remain, with 3 red ones left, so the chance the second is also red is 3/9 = 1/3, not 4/10 again. Multiplying along the branches: 4/10 × 1/3 = 2/15 for both red.

A 2023 Paper 2 'without replacement' probability question was poorly answered, with many students assuming replacement and using the same fraction for both draws. A 2022 report also noted that when a question wants a probability, giving a raw frequency (the count, not divided by the total) only earns partial credit at best. Always check whether the total changes between draws, and always express the final answer as a probability out of 1, not as a count.

Spot it Add or multiply?

A spinner is spun twice. You want the probability of getting red on the first spin AND blue on the second. What do you do with the two individual probabilities?

(A) Multiply them
(B) Add them
(C) Subtract the smaller from the larger
(D) Find their average

Cover this, choose, then read

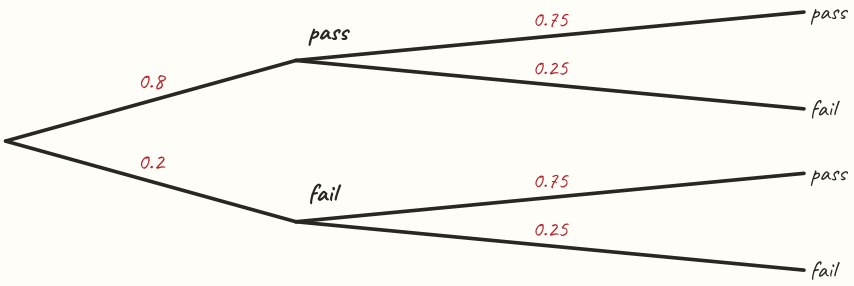
A. 'And' along a sequence of independent events means multiply the probabilities together. 'Add' is for 'or' between separate, mutually exclusive routes to the same final outcome, which isn't what's being asked here.

Part five · Probability

16 Probability

Use it

A tree diagram for two independent events



P(pass both) = 0.8 × 0.75 = 0.6. Multiply along a branch; the four end probabilities should add to 1 as a check.

A bag has 10 balls: 4 red and 6 blue. Two balls are drawn at random, without replacement. Find the probability that both are red. [3 marks]

P(first red) = 4/10 ✓. P(second red, given first was red) = 3/9 ✓. P(both red) = 4/10 × 3/9 = 12/90 = 2/15 ✓.

3/3

B1 for the first probability, 4/10. M1 for reducing both the red count and the total by 1 for the second draw, showing the dependence (without replacement). A1 for the final answer, 2/15, correctly simplified. A common error here is to use 4/10 again for the second draw, which treats dependent events as independent and loses at least the method mark.

It could be asked as

- Complete a tree diagram and use it to find the probability of a combined event.
- Find a missing probability using the fact that exhaustive outcomes sum to 1.
- Use a Venn diagram or two-way table to find a probability.
- Find the probability of a combined event without replacement, where the second probability depends on the first.
- HT Find a conditional probability using a two-way table or a Venn diagram.

Cover both pages and write your sixty seconds from memory

predict ___ / 10 got ___ / 10

Quick-fire. Answers are on page 48

16.1 A fair coin is flipped twice. Find the probability of two heads.

16.2 A spinner has P(red) = 0.3. Find the expected number of reds in 200 spins.

16.3 Outcomes A, B and C are exhaustive and mutually exclusive, with P(A) = 0.2 and P(B) = 0.45. Find P(C).

16.4 A bag has 5 red and 3 blue counters. Two are drawn without replacement. Find P(both blue).

Part six · Statistics

17 Statistics

Learn it

In sixty seconds

Describe data with an average (mean, median or mode) and a measure of spread (range, or the interquartile range). Represent data with the right chart for its type: bar charts and pie charts for categories, and at Higher tier, box plots, histograms and cumulative frequency graphs for grouped or continuous data.

Facts to know

Try it, then colour the circle: ● *knew it* ◐ *shaky* ○ *missed*

- ○ Interpret and construct **tables, charts and diagrams**: frequency tables, bar charts, pie charts and pictograms for categorical data, and vertical line charts for ungrouped discrete numerical data.
- ○ Interpret and construct tables and **line graphs for time series data** (data collected at successive points in time), and know which diagram suits which type of data.
- ○ Infer properties of a population from a sample, while knowing the limitations of sampling: a small or unrepresentative sample can give a misleading picture of the whole population.
- ○ Find and interpret measures of **central tendency**: the mean (total ÷ number of values), the median (the middle value when ordered) and the mode (the most common value), and the modal class for grouped data.
- ○ Find and interpret measures of **spread**: the range (biggest minus smallest value), including thinking about outliers that might distort it.
- ○ HT Find the **quartiles** and the **interquartile range** (IQR = upper quartile − lower quartile) of a data set, and construct and interpret a **box plot**, which shows the minimum, lower quartile, median, upper quartile and maximum.
- ○ HT Construct and interpret a **histogram**, for grouped continuous data with equal or unequal class widths, using **frequency density** = frequency ÷ class width on the vertical axis, so that the *area* of each bar represents the frequency.
- ○ HT Construct and interpret a **cumulative frequency graph**, plotting the running total of frequency against the upper class boundary, and use it to estimate the median and the quartiles.
- ○ Use and interpret **scatter graphs** of bivariate data (two related variables); recognise positive, negative, weak, strong or no correlation, and know that correlation does not mean causation.
- ○ Draw an estimated line of best fit on a scatter graph, and use it to make predictions; know the dangers of interpolating (predicting within the data range, fairly safe) versus extrapolating (predicting well beyond it, less reliable).

Key words

- **Mean** the total of all the values, divided by how many values there are. ☐
- **Median** the middle value in an ordered list of data. ☐
- **Mode** the value (or category) that occurs most often. ☐
- **Interquartile range** the range of the middle 50% of the data: upper quartile minus lower quartile. ☐
- **Frequency density** frequency divided by class width, plotted on a histogram's vertical axis. ☐
- **Cumulative frequency** the running total of frequencies, added up as you go. ☐
- **Correlation** a relationship between two variables shown on a scatter graph. ☐
- **Outlier** a value much larger or smaller than the rest of the data. ☐

Why histograms use frequency density, not frequency. If class widths are unequal, a tall bar on a plain frequency axis could just mean a wide class, not a genuinely common range of values. Frequency density (frequency ÷ width) fixes this: the *area* of each bar, not its height, then correctly represents the frequency, so bars of different widths can be compared fairly.

A 2022 histogram question found many students reading the frequency straight off the vertical (frequency density) axis instead of multiplying by the class width to recover it, and a 2022 box plot question saw an interquartile range miscalculated using the wrong operation when a scale factor was involved. Always check what each axis of a chart actually represents before reading numbers from it, and write down the calculation, not just the final figure.

Spot it Which measure is being described?

Which average is found by dividing the total of all the values by how many values there are?

(A) The median (B) The mode

(C) The mean (D) The range

Cover this, choose, then read

C. That description is exactly the definition of the mean. The median is the middle value when the data is ordered, the mode is the most frequent value, and the range is a measure of spread, not an average, so none of those fit the description given.

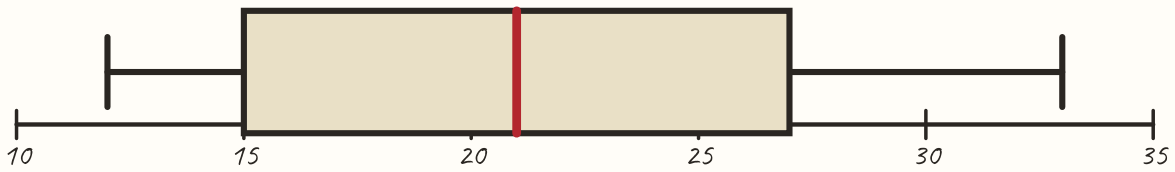
17 Statistics

Part six · Statistics

Use it

Five-number summary and a box plot, for an invented data set

Values: 12 to 33



Minimum	Lower quartile	Median	Upper quartile	Maximum
12	15	21	27	33

HT Eleven students' test scores, in order, are: 12, 15, 15, 18, 19, 21, 22, 24, 27, 30, 33. Find the median, the interquartile range, and draw a box plot. [4 marks]

Median = the 6th value (middle of 11) = 21 ✓. Lower quartile = middle of the 5 values below the median = 15 ✓.

Upper quartile = middle of the 5 values above the median = 27 ✓.

IQR = 27 − 15 = 12 ✓.

4/4

B1 for the median, 21. M1 for a correct method for the quartiles (splitting the data either side of the median into two equal halves). A1 for both quartiles, 15 and 27. A1 for the interquartile range, 12. The box plot itself should show a box from 15 to 27 with a line at 21, and whiskers out to 12 and 33.

It could be asked as

- Find the mean, median, mode or range of a data set, including from a frequency table.
- HT Find the quartiles and interquartile range, and draw or interpret a box plot.
- HT Construct or interpret a histogram with unequal class widths, using frequency density.
- Describe the correlation shown on a scatter graph, and use a line of best fit to make a prediction.

Cover both pages and write your sixty seconds from memory predict ___ / 10 got ___ / 10

Quick-fire. Answers are on page 48

17.1 Find the mean of 4, 7, 9, 12, 18.

17.2 Find the range of 23, 8, 15, 31, 19.

17.3 HT A histogram bar has frequency density 3.5 and class width 10. Find its frequency.

17.4 A scatter graph shows points rising steadily left to right. What type of correlation is this?

Skills

Which formulae you need to know

AQA's Appendix splits every GCSE Maths formula into three groups. Knowing which group a formula is in tells you exactly how hard to work at memorising it.

1. Must know, never given

These are not on the Formulae Sheet. Learn them so well you could write them from memory under exam pressure.

Quadratic formula

$x = (-b \pm \sqrt{(b^2 - 4ac)}) \div 2a$

solves $ax^2 + bx + c = 0$

Circle

$C = 2\pi r = \pi d$, $A = \pi r^2$

circumference and area

Pythagoras

$a^2 + b^2 = c^2$

right-angled triangles

Trig ratios

$\sin A = a/c$, $\cos A = b/c$, $\tan A = a/b$

right-angled triangles

HT Sine rule

$a/\sin A = b/\sin B = c/\sin C$

any triangle

HT Cosine rule

$a^2 = b^2 + c^2 - 2bc \cos A$

any triangle

HT Area of any triangle

Area = $\frac{1}{2}ab \sin C$

two sides and the included angle

Skills

Which formulae you need to know, continued

2. Must know, or be able to derive

Also not given, but these can usually be worked out quickly from first principles if you forget the exact form, such as splitting a trapezium into a rectangle and a triangle.

Trapezium area

Area = ½(a + b)h

a, b parallel sides; h the gap between them

Prism volume

Volume = area of cross-section × length

constant cross-section

Compound interest

Total = $P(1 + r/100)^n$

P principal, r rate, n time periods

Probability: or

P(A or B) = P(A) + P(B) − P(A and B)

subtract the overlap, so it isn't double-counted

HT Probability: and

P(A and B) = P(A given B) × P(B)

the conditional rule

3. Given on the Formulae Sheet

Printed on every paper's Insert. You still need to know *when* to use them and how to substitute correctly: the method mark is for using the formula properly, not for having memorised it.

Cone, curved surface

$A = \pi rl$

l is the slant height

Sphere, surface area

$A = 4\pi r^2$

Sphere, volume

$V = \frac{4}{3}\pi r^3$

Cone, volume

$V = \frac{1}{3}\pi r^2 h$

h is the perpendicular height

HT suvat: velocity

$v = u + at$

kinematics, Higher tier only

HT suvat: displacement

$s = ut + \frac{1}{2}at^2$

HT suvat: velocity²

$v^2 = u^2 + 2as$

Skills

Calculator and non-calculator skills

Paper 1 is non-calculator; Papers 2 and 3 allow one. Both need deliberate practice: non-calculator work needs reliable number facts, and a calculator needs to be used correctly, not just owned.

Without a calculator

- ☐ **Times tables to 12 × 12**, both ways: instantly knowing 7 × 8 = 56 also means instantly knowing 56 ÷ 8 = 7.
- ☐ **Fraction arithmetic**: find a common denominator to add or subtract; multiply straight across for ×; flip the second fraction and multiply for ÷.
- ☐ **Percentage of an amount**, by building from 10% and 1%: 10% of £64 is £6.40, so 5% is £3.20 and 15% is £9.60, all without a written multiplication.
- ☐ **Doubling, halving and partitioning**: 23 × 6 as (20 × 6) + (3 × 6) = 120 + 18 = 138, rather than a column method for everything.
- ☐ **Column methods** for addition, subtraction, multiplication and division, kept neat, with place value lined up.
- ☐ **Checking by estimation**: round every number to 1 significant figure first, and compare the estimate with your actual answer before moving on.

Watch out

Marks are available for method on a non-calculator paper even with a wrong final value: write every step down, including the fraction or decimal working, not just a final number.

With a calculator

- ☐ **Standard form entry**: use the calculator's $\times 10^{x}$ key, not × then 1, then 0, then the power, to avoid order-of-operations mistakes.
- ☐ **The memory function**: store an intermediate result (such as a denominator you'll need again) rather than rounding it and retyping.
- ☐ **Brackets**: for a fraction such as (5 + 7) ÷ (2 × 3), bracket the numerator and the denominator separately, or the calculator will divide by only part of it.
- ☐ **Trig and inverse trig functions**: sin, cos and tan find a ratio from an angle; $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$ (often shift + the same key) find an angle from a ratio.
- ☐ **Avoiding premature rounding**: keep a value on the calculator display (or use the memory or the ANS key) and carry it through unrounded, rounding only the final answer.
- ☐ **Fraction and power keys**: typing a fraction into the fraction key, rather than converting to a decimal first, keeps an exact answer where one is needed.

Watch out

Rounding too early is penalised even on a calculator paper. If an answer needs 3 significant figures, keep every digit the calculator gives you until the very last line of working, then round once.

Skills

What AQA's examiners saw students get wrong

These come from AQA's Reports on the Examination for the Higher papers (June 2022 and June 2023). They're paraphrased. The struck-through words are what students wrote; the green words are what actually earns the mark.

Converting to decimals. Misplacing the point, such as ~~0.035~~ instead of the correct value. Check the size of the answer makes sense against the original number.

Dividing by a fraction. ~~Forgetting to invert the second fraction~~, or inverting the wrong one. Keep the first fraction as it is; flip the second, then multiply.

Solving an inequality. Giving ~~$x = 13$~~ or a list of integers instead of the inequality itself. An inequality's answer is a range, written with the inequality sign, unless integers are specifically asked for.

Squaring a fraction. Squaring only the numerator, such as $(3/2)^2$ treated as ~~3/2~~. Square the numerator and the denominator separately: 9/4.

Prime numbers. Treating ~~27~~ as prime, or forgetting a stated condition such as 'even'. Check against the actual definition: exactly two factors, 1 and itself.

Sharing a ratio. Taking ~~one share only~~ from the total, or dividing by the wrong number of parts. Add the parts first, divide the total by that, then multiply up for each share.

Laws of indices. Multiplying the ~~bases as well as~~ adding the powers. The base stays the same; only the indices add, subtract or multiply.

Criticising a graph or statement. Vague answers such as ~~one line touches the axis and the other doesn't~~, with no named feature. Name the specific feature (an intercept, a gradient, a scale) and say exactly what is wrong with it.

Algebraic word problems. Using ~~$x + 1$~~ for 'one year younger', or forgetting to include every term in the final total. Translate the words literally: younger means subtract; check every quantity mentioned is in the final equation.

Reading probabilities. Giving a raw frequency, such as ~~41~~, instead of a probability out of the total. Always divide the count by the total number of outcomes before writing the final answer.

Error intervals and bounds. Giving a lower bound of ~~0~~ instead of half the rounding unit below the value. The bound is always half the degree of accuracy, not zero, unless the value itself is zero.

Negative indices. Linking a negative power to ~~one over the power itself~~, rather than one over the whole value. a^{-n} means $1 \div a^n$: invert the whole expression, not just the exponent.

Function machines. Substituting a specific number where the question needs a ~~general rule~~ in terms of x. If the question says 'any input' or uses a letter, the answer must use that letter too, not a chosen example.

Composite functions. Evaluating one function ~~alone~~ and stopping, without substituting the result into the second. fg(x) means work out g(x) first, then put that whole result into f.

Quartiles. Assuming the median is always exactly ~~double~~ the lower quartile, without checking the real data. Find the lower quartile and the median separately from the ordered list; don't assume a relationship between them.

Simultaneous equations. Subtracting when matching coefficients have ~~different signs~~ (or adding when they're the same). Same signs: subtract to eliminate. Different signs: add to eliminate.

Surface area of a composite solid. Leaving out a ~~flat face~~ created where two shapes are joined together. Sketch the solid first and tick off every face, including any new flat faces made by the join.

Histograms. Reading the ~~frequency density~~ value straight off the axis as if it were the frequency itself. Multiply frequency density by the class width to recover the actual frequency.

Bearings. Measuring the wrong angle, such as ~~an anticlockwise or reflex angle~~, instead of a true clockwise bearing from north. Always measure clockwise from north, and give the bearing in three figures, such as 045°.

Compound interest. Using ~~simple interest~~, multiplying one year's growth by the number of years, instead of compounding. Raise the multiplier to the power of the number of years: $P \times (1 + r/100)^n$ in a single calculation.

Describing a transformation. Describing it as ~~a combination~~ of two transformations, which earns no marks for a 'describe' question. Name exactly one transformation, with every piece of information it needs (such as the mirror line, or the centre and scale factor).

Circle theorems. Stating an angle's value ~~without naming~~ which circle theorem justifies it. Name the theorem in words every time you use one, not just the number it produces.

Right-angled trigonometry. Assigning the opposite and adjacent sides to the ~~wrong angle~~ in a diagram with more than one triangle. Label which angle you're using in each triangle before deciding which ratio applies.

Vectors. Leaving the brackets off a column vector, or drawing a ~~line between the top and bottom numbers~~ like a fraction. A column vector is always written in brackets, one number above the other, with no dividing line.

Skills

A problem-solving question, marked at three levels

AQA's longer problem-solving questions are marked with a mix of M (method), A (accuracy) and B (independent) marks. Seeing the same question answered at three different standards shows exactly where the marks really sit, and where they're lost.

A garden path is 12 m long and 1.5 m wide. Paving costs £18.50 per m², plus a flat delivery charge of £45. The homeowner has a budget of £400. Is this enough? Show your working, and state how much is left over or how much more is needed. [5 marks]

Area = 12 × 1.5 = 18 m² ✓. Cost of paving = 18 × £18.50 = £333 ✓. Total cost = £333 + £45 = £378 ✓, confirmed ✓. £378 is less than £400, so yes, the budget is enough, with £400 − £378 = £22 left over ✓.

5/5

M1 area. M1 paving cost. M1 add delivery. A1 correct total, £378. B1 (dep on A1) a clear decision with the correct amount left over.

The same question at three levels

Level 1 (1 to 2 marks). "12 × 1.5 = 18. 18 × 18.50 = 333." The area and the paving cost are both right (two of the three method marks), but the delivery charge is never added, and there is no total and no decision, so the three final marks (the third M1, the A1 and the B1) are all unavailable.

Level 2 (3 to 4 marks). "12 × 1.5 = 18. 18 × 18.50 = 333. 333 + 45 = 378." Every calculation is correct and complete, earning all three M marks and the A1 for £378, but there is no final sentence saying whether £400 is enough: without that decision, the dependent B1 is not awarded, even though every number is right.

Level 3 (5 marks). The full answer above: the calculation, the correct total, and a decision that states both the comparison and the amount left over.

The lesson

Two of the three attempts above get every single number right. Only the version that finishes with an explicit, correctly reasoned decision scores full marks. A correct calculation that stops short of answering the actual question asked is a very common way to drop the last one or two marks on this question type.

Skills

A second problem-solving question, marked at three levels

This one needs an equation to be set up from words, not just solved. Forming the equation correctly is usually worth as many marks as solving it.

The length of a rectangle is 3 cm more than its width, x cm. The area of the rectangle is 70 cm². Form an equation in x, solve it, and state the width and the length of the rectangle. [6 marks]

Length = x + 3 ✓. Area: x(x + 3) = 70 ✓. Expanding: $x^2 + 3x - 70 = 0$ ✓. Factorising: (x + 10)(x − 7) = 0 ✓, so x = -10 or x = 7; reject x = -10 since a width can't be negative ✓. Width = 7 cm. Length = 7 + 3 = 10 cm ✓.

6/6

M1 forming x(x + 3) = 70. M1 rearranging to $x^2 + 3x - 70 = 0$. M1 a correct method to solve the quadratic. A1 x = 7, with -10 rejected. B1 width stated as 7 cm. B1 (dep) length stated as 10 cm.

The same question at three levels

Level 1 (1 to 2 marks). "Width could be about 7, because 7 × 10 = 70." No equation is ever formed and no algebra is shown, so even though the final numbers happen to be correct, only the first method mark for an equivalent correct equation might be inferred, and often not even that: a guess that happens to work is not the same as the method AQA is testing.

Level 2 (3 to 4 marks). "x(x + 3) = 70, so $x^2 + 3x - 70 = 0$. Using the quadratic formula, x = (-3 ± √(9 + 280)) ÷ 2." All three method marks are earned for forming and rearranging the equation and attempting a correct solving method, but a slip with the square root (√289 mistaken for a non-whole number) leads to a rounded, incorrect value for x, losing the A1 and both B marks that depend on it.

Level 3 (6 marks). The full answer above: a correctly formed and solved equation, the negative root rejected for a sound reason, and both the width and the length stated clearly.

The lesson

A numerically correct final answer reached by guessing, rather than by forming and solving the equation the question actually asks for, scores close to nothing. Show the equation, show the solving method, and always check whether a negative or otherwise impossible root needs to be rejected and said so.

Practice

Practice paper, part one

67 marks over four pages, about 74 minutes. Written for this guide, with invented numbers throughout. Mark it on pages 48–50. Try part one without a calculator, the way Paper 1 works; a green **Calculator** tag marks a question meant for Papers 2 or 3. HT marks Higher tier questions.

1 Tick one box for each question. [1 mark each] *[4]*

1.1 Which of these numbers is the smallest?

- ☐ A -3
- ☐ B 0.2
- ☐ C -1/2
- ☐ D 1.5

1.2 Simplify $4^5 \div 4^2$

- ☐ A 4^3
- ☐ B 4^7
- ☐ C 4^{10}
- ☐ D 16^3

1.3 Which is the correct expansion of $3(2x - 1)$?

- ☐ A $6x - 1$
- ☐ B $6x - 3$
- ☐ C $5x - 1$
- ☐ D $6x + 3$

1.4 Which decimal equals 7/8?

- ☐ A 0.78
- ☐ B 0.875
- ☐ C 0.87
- ☐ D 0.8

2 Work out 3¼ + 1⅚, giving your answer as a mixed number. *[3]*

3 Write 56 as a product of its prime factors. *[2]*

4.1 Factorise $x^2 - 5x - 24$. *[2]*

4.2 Find the nth term of the sequence 4, 9, 14, 19, … *[2]*

5 Solve $4(x + 3) = 2x + 18$. *[3]*

6 **HT** n is an integer. Prove algebraically that $(n + 1)^2 - (n - 1)^2$ is always a multiple of 4. *[4]*

Practice

Practice paper, part two

7 Share £350 in the ratio 2 : 3 : 5. *[3]*

8 Calculator A map has a scale of 1 : 25000. Two towns are 8.4 cm apart on the map. Work out the real distance between them, in km. *[3]*

9 HT Calculator y is inversely proportional to x^2. When x = 2, y = 15.
HT (a) Find y when x = 5.
HT (b) Find x when y = 0.6. *[4]*

10 Calculator The exterior angle of a regular polygon is 24°. Work out the number of sides, and the size of each interior angle. *[3]*

11.1 Calculator A ship sails from port A on a bearing of 072° for 15 km to point B. Work out, to the nearest km, the ship's distance east of A. *[3]*

11.2 Calculator A circular pond of radius 3 m is surrounded by a path 1.2 m wide. Work out the area of the path, to the nearest m^2. *[3]*

Practice

Practice paper, part three

12 HT Calculator A triangle has sides 7 cm and 10 cm with an included angle of 48°. Find its area, to 1 decimal place. (Area = ½ab sin C) *[3]*

13 HT Calculator A triangle has sides a = 9, b = 11 and c = 13 (opposite angles A, B, C). Find the size of angle C, to 1 decimal place. (Cosine rule) *[4]*

14 HT A, B and C are points on a circle with centre O. Angle AOC at the centre is 136°. B is on the major arc AC. Find the size of angle ABC, giving a reason. (Circle theorem) *[3]*

15 Calculator A box has 9 counters: 5 red and 4 blue. Two counters are drawn at random, without replacement.
(a) Find the probability both are red.
(b) Find the probability of exactly one red. *[4]*

Practice

Practice paper, part four

16.1 HT A class of continuous data, $20 \leq x < 35$, has frequency 45. Find the frequency density. *[2]*

16.2 HT A second class, $35 \leq x < 40$, has frequency density 2.4. Find its frequency. *[2]*

17 HT Eleven students' scores, in order, are: 5, 7, 7, 9, 11, 12, 14, 16, 18, 20, 23. Find the median, the lower and upper quartiles, and the interquartile range. *[4]*

18 Calculator A garden centre buys trees at £24 each and sells them at a 65% mark-up. In a sale, the price is reduced by 20%. A customer buys 3 trees in the sale, and gets free delivery (normally £15) only if they spend over £100 in the sale. Work out how much the customer pays in total, showing whether they qualify for free delivery. *[6]*

Answers

Quick-fire answers

1 Number: structure and calculation

1.1 $60 = 2^2 \times 3 \times 5$.

1.2 1/16.

1.3 4 (the cube root of 8 is 2, then squared is 4).

1.4 4.7×10^{-4}.

2 Fractions, decimals, percentages and accuracy

2.1 3/5.

2.2 4.8.

2.3 $11.5 \leq x < 12.5$.

2.4 $60 \times 0.2 = 12$.

3 Expressions, formulae and identities

3.1 $4x + 11$.

3.2 $(x + 4)(x - 4)$.

3.3 $r = \sqrt{(A \div \pi)}$.

3.4 $g(2) = 3$, so $fg(2) = 3^2 = 9$.

4 Straight-line graphs

4.1 Gradient -2, y-intercept 5.

4.2 Yes, both have gradient 5.

4.3 -1/2.

4.4 $y = 3x + 4$.

5 Quadratic and other graphs

5.1 One (a repeated root).

5.2 $x = 0$ and $y = 0$.

5.3 $(x + 2)^2 - 3$.

5.4 An exponential growth curve, rising steeply and never reaching 0.

6 Solving equations and inequalities

6.1 $x = 5$.

6.2 $x = 4$.

6.3 $x = 3$ or $x = -4$.

6.4 $x \geq 4$, a closed circle at 4 with the arrow pointing right.

7 Sequences

7.1 18, 22 (add 4 each time).

7.2 $3n + 2$.

7.3 No: $4n + 1 = 50$ gives $n = 12.25$, not a whole number.

7.4 $n^2 + 1$ (second difference 2, and n^2 gives 1,4,9,16,25, each 1 less).

8 Ratio and proportion

8.1 3 : 4.

8.2 £20 and £30.

8.3 $k = 5$, so $y = 35$.

8.4 $k = 24$, so $y = 8$.

9 Percentages and financial maths

9.1 £64.80 (60×1.08).

9.2 £50 ($45 \div 0.9$).

9.3 £60 ($500 \times 0.03 \times 4$).

9.4 £2205 (2000×1.05^2).

10 Compound measures and rates of change

10.1 50 mph.

10.2 2.5 g/cm^3.

10.3 15000 N/m^2 (15000 Pa).

10.4 The instantaneous rate of change at that exact point.

11 Angles, polygons, constructions, bearings and loci

11.1 $x = 36$ ($5x = 180$).

11.2 40° ($360 \div 9$).

11.3 140° ($180 - 40$).

11.4 The perpendicular bisector of the line segment joining them.

12 Perimeter, area and volume

12.1 28 cm^2 ($\frac{1}{2} \times 14 \times 4$).

12.2 10π cm.

12.3 126 cm^3.

12.4 16π cm^2 (a quarter of $\pi \times 8^2$).

13 Pythagoras and trigonometry

13.1 10 cm.

13.2 ½.

13.3 28.3 cm^2 ($\frac{1}{2} \times 7 \times 9 \times \sin 64°$).

13.4 When you know two sides and the angle between them, or all three sides.

14 Transformations, congruence and similarity

14.1 The angle of rotation and its direction (clockwise or anticlockwise).

14.2 16 (4^2).

14.3 125 (5^3).

14.4 SAS.

15 Circle theorems and vectors

15.1 70° ($180 - 110$).

15.2 90°.

15.3 (8, 4).

15.4 They are parallel (CD is a scalar multiple of AB).

16 Probability

16.1 ¼ ($\frac{1}{2} \times \frac{1}{2}$).

16.2 60 (0.3×200).

16.3 0.35 ($1 - 0.2 - 0.45$).

16.4 $3/8 \times 2/7 = 3/28$.

17 Statistics

17.1 10 (total $50 \div 5$).

17.2 23 ($31 - 8$).

17.3 35 (3.5×10).

17.4 Positive correlation.

Answers

Your tracker

Colour a circle when it's done. Return two days later, and a week later.

Topic	Learn it	Use it	Recall box	Quick-fire	2 days later	1 week later
1 Number: structure and calculation	○	○	○	○	○	○
2 Fractions, decimals, percentages and accuracy	○	○	○	○	○	○
3 Expressions, formulae and identities	○	○	○	○	○	○
4 Straight-line graphs	○	○	○	○	○	○
5 Quadratic and other graphs	○	○	○	○	○	○
6 Solving equations and inequalities	○	○	○	○	○	○
7 Sequences	○	○	○	○	○	○
8 Ratio and proportion	○	○	○	○	○	○
9 Percentages and financial maths	○	○	○	○	○	○
10 Compound measures and rates of change	○	○	○	○	○	○
11 Angles, polygons, constructions, bearings and loci	○	○	○	○	○	○
12 Perimeter, area and volume	○	○	○	○	○	○
13 Pythagoras and trigonometry	○	○	○	○	○	○
14 Transformations, congruence and similarity	○	○	○	○	○	○
15 Circle theorems and vectors	○	○	○	○	○	○
16 Probability	○	○	○	○	○	○
17 Statistics	○	○	○	○	○	○

Lost marks on the practice paper? Go here

Question	Go to	Pages
1.1, 1.2, 3	Number toolkit	4–5
1.4, 2	Fractions, decimals, percentages and accuracy	6–7
1.3, 4.1, 6	Expressions, formulae and identities	8–9
4.2	Sequences	16–17
5	Solving equations and inequalities	14–15
7, 8, 9	Ratio and proportion	18–19
10, 11.1	Angles, polygons, constructions, bearings and loci	24–25
11.2	Perimeter, area and volume	26–27
12, 13	Pythagoras and trigonometry	28–29
14	Circle theorems and vectors	32–33
15	Probability	34–35
16, 17	Statistics	36–37
18	Percentages and financial maths	20–21

Answers

Practice paper: marks and answers

1 1.1 A, -3 (the most negative value is the smallest). 1.2 A, 4^3 (subtract the powers). 1.3 B, 6x − 3. 1.4 B, 0.875. [1 mark each]

2 Common denominator 12: 39/12 + 22/12 [1] = 61/12 [1] = 5 1/12 [1].

3 56 = 2 × 28 = 2 × 2 × 14 = 2 × 2 × 2 × 7 [1] = $2^3 \times 7$ [1].

4.1 Two numbers that multiply to -24 and add to -5: -8 and 3 [1]. (x − 8)(x + 3) [1].

4.2 Common difference 5 [1]. nth term = 5n − 1 [1].

5 4x + 12 = 2x + 18 [1]. 2x = 6 [1]. x = 3 [1].

6 HT $(n + 1)^2 - (n - 1)^2 = (n^2 + 2n + 1)$ [1] − $(n^2 - 2n + 1)$ [1] = 4n [1], which is a multiple of 4 for every integer n [1].

7 2 + 3 + 5 = 10 parts [1]. One part = 350 ÷ 10 = £35 [1]. Shares: £70, £105, £175 [1].

8 8.4 × 25000 = 210000 cm [1]. 210000 ÷ 100 ÷ 1000 [1] = 2.1 km [1].

9 HT $k = 15 \times 2^2 = 60$ [1]. (a) $y = 60 \div 5^2 = 2.4$ [1]. (b) $x^2 = 60 \div 0.6 = 100$ [1], so x = 10 [1].

10 360 ÷ 24 = 15 sides [1, 1]. Interior angle = 180 − 24 = 156° [1].

11.1 East component = 15 × sin 72° [1] = 14.27... [1] = 14 km (nearest km) [1].

11.2 Outer radius = 3 + 1.2 = 4.2 m [1]. Area = $\pi(4.2^2 - 3^2) = \pi \times 8.64$ [1] = 27 m^2 (nearest m^2) [1].

12 HT Area = ½ × 7 × 10 × sin 48° [1] = 35 × 0.743... [1] = 26.0 cm^2 (1 d.p.) [1].

13 HT cos C = $(9^2 + 11^2 - 13^2) \div (2 \times 9 \times 11)$ [1] = 33 ÷ 198 [1] = 0.167 [1]. C = $\cos^{-1}(0.167)$ = 80.4° (1 d.p.) [1].

14 HT The angle at the centre is twice the angle at the circumference, subtended by the same arc [1]. Angle ABC = 136° ÷ 2 [1] = 68° [1].

15 (a) P(both red) = 5/9 × 4/8 [1] = 5/18 [1]. (b) P(exactly one red) = (5/9 × 4/8) + (4/9 × 5/8) [1] = 20/72 + 20/72 = 5/9 [1].

16 HT 16.1 Frequency density = 45 ÷ 15 [1] = 3 [1]. 16.2 Frequency = 2.4 × 5 [1] = 12 [1].

17 HT Median (6th of 11) = 12 [1]. Lower quartile (middle of the lowest 5) = 7 [1]. Upper quartile (middle of the highest 5) = 18 [1]. Interquartile range = 18 − 7 = 11 [1].

18 Sale price per tree: 24 × 1.65 = £39.60 [1], then × 0.8 = £31.68 [1]. Three trees: 31.68 × 3 = £95.04 [1, 1]. £95.04 is less than £100, so free delivery does not apply [1]. Total = £95.04 + £15 = £110.04 [1].

Mark with a different colour and add up. Your teacher may accept other correct methods and wording.

More from Empower Educators

The Maths range

If this guide was useful, here is the rest of the Maths shelf. Everything shown is on the site now.

Schemes of work

Year 10 to Year 11. One sequence table a year, unit by unit and in teaching order.

Knowledge organisers

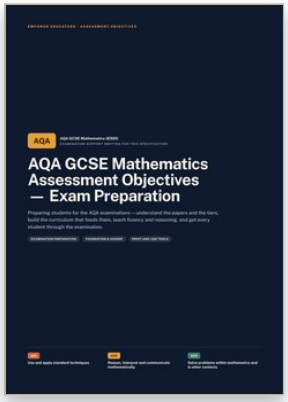
Year 7 to Year 11. A4, printable, one per unit.

GCSE questions and answers

100 exam-style questions, marked M1/A1/B1 exactly as AQA's own mark schemes do.

GCSE knowledge mats

One landscape sheet a topic, such as algebra and graphs, or geometry and trigonometry.



GCSE exam board playbook

Command words, paper structure and formulae for AQA, in one book.

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The core pure maths ideas beside a worked example and the marks examiners actually give.

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